José Luis **Díaz-Barrero**
Barcelona Mathematical Circle (BMC)
jose.luis.diaz@upc.edu

UP.625. *Proposed by Daniel Sitaru, Romania.* Find:

$$\iint_D (3x+2y)\,dx\,dy$$

where D is the interior of triangle AOB with vertices:

$$O(0,0), \quad A(2,4), \quad B(5,4)$$

Solution by José Luis Díaz-Barrero, Barcelona, Spain. To evaluate the double integral

$$\iint_D (3x+2y)\,dx\,dy$$

where D is the interior of triangle AOB with vertices $O(0,0)$, $A(2,4)$, and $B(5,4)$, we first determine the boundaries of the region. The triangle is bounded by three lines:

- Line OA: Passes through $(0,0)$ and $(2,4)$. Equation: $y = 2x \Rightarrow x = \dfrac{y}{2}$
- Line OB: Passes through $(0,0)$ and $(5,4)$. Equation: $y = \frac{4}{5}x \Rightarrow x = \dfrac{5}{4}y$
- Line AB: Horizontal line at $y = 4$.For a fixed y between 0 and 4, x ranges from the left boundary $x = \dfrac{y}{2}$ to the right boundary $x = \dfrac{5}{4}y$.

The integral becomes

$$\int_0^4 \int_{y/2}^{5y/4} (3x+2y)\,dx\,dy$$

Integrating with respect to x yields

$$\int_{y/2}^{5y/4} (3x+2y)\,dx = \left[\frac{3}{2}x^2 + 2yx\right]_{y/2}^{5y/4}$$

$$= \left(\frac{3}{2}(5y/4)^2 + 2y(5y/4)\right) - \left(\frac{3}{2}\left(\frac{y}{2}\right)^2 + 2y\left(\frac{y}{2}\right)\right) = \frac{111}{32}y^2$$

Now, integrating with respect to y, we obtain

$$\int_0^4 \frac{111}{32}\,y^2\,dy = \frac{111}{32}\left[\frac{y^3}{3}\right]_0^4 = \frac{111}{32}\left(\frac{64}{3}\right) = 74,$$

and

$$\iint_D (3x+2y)\,dx\,dy = 74.$$