

ROMANIAN MATHEMATICAL MAGAZINE

UP.620 Find the curvilinear integral of the first kind:

$$\Omega = \int_{\gamma} z \, ds; \gamma: \begin{cases} x = t \cos t \\ y = t \sin t; t \in [0, \pi] \\ z = \sqrt{3}t \end{cases}$$

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Solution by proposer

$$x(t) = t \cos t \Rightarrow x'(t) = \cos t - t \sin t$$

$$y(t) = t \sin t \Rightarrow y'(t) = \sin t + t \cos t$$

$$z(t) = \sqrt{3}t \Rightarrow z'(t) = \sqrt{3}$$

$$(x'(t))^2 + (y'(t))^2 + (z'(t))^2 = (\cos t - t \sin t)^2 + (\sin t + t \cos t)^2 + (\sqrt{3})^2 =$$

$$= \cos^2 t + t^2 \sin^2 t - 2t \sin t \cos t + \sin^2 t + t^2 \cos^2 t + 2t \sin t \cos t + 3 =$$

$$= (\sin^2 t + \cos^2 t) + t^2(\sin^2 t + \cos^2 t) + 3 = t^2 + 4$$

$$ds = \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2} dt = \sqrt{t^2 + 4} dt$$

$$\Omega = \int_{\gamma} z \, ds = \int_0^{\pi} \sqrt{3} t \cdot \sqrt{t^2 + 4} dt = \sqrt{3} \int_0^{\pi} t \cdot (t^2 + 4)^{\frac{1}{2}} dt =$$

$$= \frac{\sqrt{3}}{2} \int_0^{\pi} (t^2 + 4)' (t^2 + 4)^{\frac{1}{2}} dt = \frac{\sqrt{3}}{2} \cdot \frac{(t^2 + 4)^{\frac{3}{2}}}{\frac{3}{2}} \Big|_0^{\pi} =$$

$$= \frac{\sqrt{3}}{3} \left((\pi^2 + 4)^{\frac{3}{2}} - 4^{\frac{3}{2}} \right) = \frac{\sqrt{3}}{3} \left((\pi^2 + 4) \sqrt{\pi^2 + 4} - 8 \right)$$