ROMANIAN MATHEMATICAL MAGAZINE

UP.619 Find the curvilinear integral of the second kind:

$$\int_C yz\,dx + xz\,dy + xy\,dz$$

$$C: \begin{cases} x(t) = t \\ y(t) = 2t\,;\, t \in [0,1] \\ z(t) = 3t \end{cases}$$

Proposed by Daniel Sitaru – Romania

Solution 1 by proposer

$$P\big(x(t); y(t); z(t)\big) = y(t)z(t) = 2t \cdot 3t = 6t^2$$

$$Q\big(x(t); y(t); z(t)\big) = x(t)z(t) = t \cdot 3t = 3t^2$$

$$R\big(x(t); y(t); z(t)\big) = x(t)y(t) = t \cdot 2t = 2t^2$$

$$x'(t) = 1; y'(t) = 2; z'(t) = 3$$

$$\int_C yz\,dx + xz\,dy + xy\,dz = \int_0^1 (6t^2 \cdot 1 + 3t^2 \cdot 2 + 2t^2 \cdot 3)\,dt =$$

$$= 18\int_0^1 t^2\,dt = 18 \cdot \frac{t^3}{3}\bigg|_0^1 = \frac{18}{3} = 6$$

Solution 2 by Jose Luis Diaz Barrero-Spain

1. Computing the differentials, we have $dx = dt, dy = 2dt, dz = 3dt$.

2. Substituting the parametric equations into the integral components, we get

$$yzdx = (2t)(3t)dt = 6t^2dt$$

$$xzdy = (t)(3t)(2dt) = 3t^2(2dt) = 6t^2dt$$

$$xydz = (t)(2t)(3dt) = 2t^2(3dt) = 6t^2dt$$

3. Adding up the components and integrating from 0 to 1 yields

$$\int_C yz\,dx + xz\,dy + xy\,dz = \int_0^1 (6t^2 + 6t^2 + 6t^2)\,dt =$$

$$= \int_0^1 18t^2\,dt = 18 \cdot \frac{t^3}{3}\bigg|_0^1 = \frac{18}{3} = 6$$