UP.618 Find the curvilinear integral of the second kind:

$$\int_C (y+z)\,dx + (x+z)dy + (x+y)dz$$

$$C:\begin{cases} x(t) = t+1 \\ y(t) = t+2\,;\, t \in [0,1] \\ z(t) = t+3 \end{cases}$$

Proposed by Daniel Sitaru – Romania

Solution 1 by proposer

$$P\big(x(t),y(t),z(t)\big) = y(t)+z(t) = t+2+t+3 = 2t+5$$

$$Q\big(x(t),y(t),z(t)\big) = x(t)+z(t) = t+1+t+3 = 2t+4$$

$$R\big(x(t),y(t),z(t)\big) = x(t)+y(t) = t+1+t+2 = 2t+3$$

$$\int_C (y+z)\,dx + (x+z)dy + (x+y)dz =$$

$$= \int_0^1 \big((2t+5)\cdot 1 + (2t+4)\cdot 1 + (2t+3)\cdot 1\big)\,dt =$$

$$= \int_0^1 (6t+13)dt = 6\cdot \frac{t^2}{2}\bigg|_0^1 + 12t|_0^1 = 3+12 = 15$$

Solution 2 by Jose Luis Diaz Barrero-Spain

$$\int_C (y+z)\,dx + (x+z)dy + (x+y)dz$$

where C is given by the parametric equations:

$$\begin{cases} x(t) = t+1 \\ y(t) = t+2 \quad \text{for } t \in [0,1] \\ z(t) = t+3 \end{cases}$$

1. Computing the differentials, we have $dx = dy = dz = dt$.

2. Substituting the parametric equations into the integral components, we get

$$y+z = (t+2)+(t+3) = 2t+5$$

$$x+z = (t+1)+(t+3) = 2t+4$$

$$x+y = (t+1)+(t+2) = 2t+3$$

3. Summing the components and integrating from 0 to 1 yields

$$\int_C (y+z)\,dx + (x+z)dy + (x+y)dz = \int_0^1 [(2t+5)+(2t+4)+(2t+3)]\,dt$$

$$= \int_0^1 (6t+12)\,dt = [3t^2+12t]_0^1 = 15$$

and we are done.