Number 42 AUTUMN 2026

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ROMANIAN MATHEMATICAL MAGAZINE

Founding Editor
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Available online
www.ssmrmh.ro

ISSN-L 2501-0099

PROBLEMS FOR JUNIORS

JP.616. Determine the largest positive value of the constant k such that:

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{d^2}-4\geq k(a+b+c+d-4)$$

for any positive real numbers a,b,c,d satisfying $ab+bc+cd+da=4$.

Proposed by Vasile Cîrtoaje - Romania

JP.617. Let a,b,c,d be nonnegative real numbers such that $a\geq b\geq c\geq d$ and $(a+d)(b+c)=4$. Prove that:

$$\frac{1}{a+3}+\frac{1}{b+3}+\frac{1}{c+3}+\frac{1}{d+3}\geq 1.$$

Proposed by Vasile Cîrtoaje - Romania

JP.618. Let $x_1,x_2,\ldots,x_m>0$, where $m\geq 2$, and use the cyclic convention $x_{m+1}=x_1$. If $\alpha>1$ and $\lambda\geq 1$, prove that:

$$\sum_{k=1}^{m}\frac{x_k((\alpha-1)x_k+2k_{k+1})}{\alpha x_k^2+x_{k+1}^2}+\lambda\sum_{k=1}^{m}\frac{(x_k-x_{k+1})^2}{x_k^2+x_{k+1}^2}\geq m$$

and determine all equality cases.

Proposed by Mihály Bencze - Romania

JP.619. Let $a_1,a_2,\ldots,a_n$ be nonzero real numbers such that $a_1+a_2+a_n=1$. Find the minimum value of

$$\sum_{k=1}^{n}\frac{a_k^3}{a_k^2+a_ka_{k+1}+a_{k+1}^2}$$

where the subscripts are taken modulo n.

Proposed by José Luis Díaz - Barrero - Spain

JP.620. Determine the number of positive integers a such that $a|12!$, but 18 does not divide a.

Proposed by José Luis Díaz - Barrero - Spain

JP.621. Determine all prime numbers p, q, r such that $p^2 + q^2 - r(p+q) = 108$

Proposed by José Luis Díaz - Barrero - Spain

JP.622. Let a, b, c be positive real such that $a+b+c=3$. Find the minimum value of

$$\frac{a^2-ab+b^2}{b+c}+\frac{b^2-bc+c^2}{c+a}+\frac{c^2-ca+a^2}{a+b}$$

Proposed by José Luis Díaz - Barrero - Spain

JP.623. Find all values of n in the range $1 \le n \le 10$ for which the sum

$$S=\sum_{k=1}^{n^2+n}\lfloor\sqrt{k}\rfloor$$

is a prime number. Here $\lfloor x \rfloor$ represents the integer part of x.

Proposed by José Luis Díaz - Barrero - Spain

JP.624. If $a, b, c > 0, a+b+c=3$ and $0 \le \lambda \le 2$ then:

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{\lambda}{abc}\le 1$$

Proposed by Marian Marin Chirciu - Romania

JP.625. If $a, b, c \ge 0, a+b+c=2$ and $\lambda \ge 1$ then:

$$\sum\sqrt{\frac{b+c}{a^2+\lambda}}\ge\frac{4}{\sqrt{\lambda}+1}$$

Proposed by Marin Chirciu - Romania

JP.626. If $a, b, c > 0, a^n+b^n+c^n=3$ and $n \in \mathbb{N}^*$ then:

$$\sum\frac{a}{a^{2n+1}+b^{2n}+c^{2n-1}}\le 1$$

Proposed by Marin Chirciu - Romania

JP.627. If $a, b, c > 0$ and $\lambda \ge 0$ then:

$$\sum\frac{\sqrt{a^2+(b+\lambda c)^2}}{b+\lambda c}\ge\frac{6\lambda+9}{(\lambda+1)\sqrt{5}}$$

Proposed by Marin Chirciu - Romania

JP.628. How many ways are there to put 15 balls into four bins so that the first two bins have between 2 and 6 balls, the third bin has at least 5 balls and the last bin has an odd number of balls?

Proposed by José Luis Díaz - Barrero - Spain

JP.629. The numbers $1, 2, \dots, 1000$ are written on the board. Two numbers are erased and replaced by the remainder of their sum when divided by 19. This operation is repeated until one number is left. What is the number?

Proposed by José Luis Díaz - Barrero - Spain

JP.630. In Imaginaryland there are 2027 towns. Every pair of towns is either connected by a single road, or is not connected. If we consider any subset of 2025 towns, the total number of roads connecting these towns to each other is a constant. If there are R roads in Imaginaryland, then find all possible values of R.

Proposed by José Luis Díaz - Barrero - Spain

PROBLEMS FOR SENIORS

SP.616. Let a, b, c, d be nonnegative real numbers such that at most one of the is less than 1 and $ab + ac + ad + bc + bd + cd = 6$. Prove that:

$$(2a^2 + 1)(2b^2 + 1)(2c^2 + 1)(2d^2 + 1) \geq 81$$

Proposed by Vasile Cîrtoaje - Romania

SP.617. Let a, b, c, d be positive real numbers such that $a \geq b \geq 1 \geq c \geq d$ and $ab + ac + ad + bc + bd + cd = 6$. Prove that:

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + 8 \geq 3(a + b + c + d)$$

Proposed by Vasile Cîrtoaje - Romania

SP.618. Prove that $\frac{17}{3}$ is the smallest positive value of the constant k such that:

$$\frac{1}{a+k} + \frac{1}{b+k} + \frac{1}{c+k} + \frac{1}{d+k} \leq \frac{4}{1+k}$$

for any $a, b, c, d \geq \frac{1}{16}$ with $ab + ac + ad + bc + bd + cd = 6$.

Proposed by Vasile Cîrtoaje - Romania

SP.619. Let a, b, c, d, e be positive real numbers such that $a \geq b \geq c \geq d \geq e$ and $abcde = 1$. Prove that:

$$\frac{1}{3ab+2}+\frac{1}{3bc+2}+\frac{1}{3cd+2}+\frac{3}{3de+2}+\frac{1}{3ea+2} \geq 1$$

Proposed by Vasile Cîrtoaje - Romania

SP.620. Let a, b, c be nonnegative real numbers such that at most one of them is larger than 1 and $ab + bc + ca = 3$. Prove that:

$$8(\sqrt{1+3a^2}+\sqrt{1+3a^2}+\sqrt{1+3a^2}) \leq 21 + 3(a+b+c)^2$$

Proposed by Vasile Cîrtoaje - Romania

SP.621. Prove that $\frac{5}{4}$ is the least positive value of the power k such that the inequality

$$(ab)^k + (ac)^k + (ad)^k + (bc)^k + (bd)^k + (cd)^k \geq 6$$

holds for any nonnegative real numbers a, b, c, d with $a \geq b \geq c \geq 1 \geq d$ and

$$ab + bc + cd + da = 4$$

Proposed by Vasile Cîrtoaje - Romania

SP.622. Compute the value of the integral:

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\cos t}{\pi^{\frac{1}{t}}+1} dt$$

Proposed by José Luis Díaz - Barrero - Spain

SP.623. Compute the following limit:

$$\lim_{n\to\infty} \frac{1}{n^2} \sum_{k=1}^{n} k\left(\frac{2k^2+3n^2}{3k^2+2n^2}\right)$$

Proposed by José Luis Díaz - Barrero - Spain

SP.624. Let ABC be a triangle. Prove that:

$$\left(\frac{1}{\sin^2 A}+\frac{1}{\sin^2 B}+\frac{1}{\sin^2 C}\right)\left(\sin^2\frac{A}{2}+\sin^2\frac{B}{2}+\sin^2\frac{C}{2}\right) \geq 3$$

Proposed by José Luis Díaz - Barrero - Spain

SP.625. If $n \geq 3$ is an integer, then prove that $\sqrt{n+2}+\sqrt{n-2}$ is an irrational number

Proposed by José Luis Díaz - Barrero - Spain

SP.626. If $a,b,c>0, a+b+c=1$ and $\lambda\ge 1$ then:

$$\sum\frac{a+\sqrt{bc}}{a+\lambda}\le\frac{1+3\lambda}{4}+\frac{1}{\lambda+1}\sum\sqrt{bc}$$

Proposed by Marin Chirciu - Romania

SP.627. Solve for reals:

$$\sqrt[3]{(1+x)^2}+\sqrt[3]{(1-x)^2}=\sqrt[3]{1+x}+\sqrt[3]{1-x}$$

Proposed by Nguyen Hung Cuong - Vietnam

SP.628. If $a,b,c>0; (a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=7$ then:

$$(a^2+b^2+c^2)\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)\ge 3$$

Proposed by Daniel Sitaru - Romania

SP.629. If $a,b>0; a+b\le 2$ then:

$$\left(\frac{2+a}{1+a}+\frac{1-2b}{1+2b}\right)\left(\frac{2+3a}{1+3a}+\frac{1-6b}{1+6b}\right)\ge\frac{64}{105}$$

Proposed by Daniel Sitaru - Romania

SP.630. In ΔABC the following relationship holds:

$$\frac{a^2}{h_b^3}+\frac{b^2}{h_c^3}+\frac{c^2}{h_a^3}\ge\frac{8}{3R}$$

Proposed by Nguyen Hung Cuong - Vietnam

UNDERGRADUATE PROBLEMS

UP.616. Calculate the following integral:

$$\int_0^\infty\frac{\ln x}{(x+1)(x^4+x^2+1)}dx$$

Proposed by Vasile Mircea Popa - Romania

UP.617. Calculate the integral:

$$\int_{-1}^{1}\frac{x^2\arccos(x)}{\sqrt{1-x^6}}dx$$

Proposed by Vasile Mircea Popa - Romania

UP.618. Find the curvilinear integral of the second kind:

$$\int_C (y+z)dx+(x+z)dy+(x+y)dz, \qquad C: \begin{cases} x(t)=t+1 \\ y(t)=t+2 \\ z(t)=t+3 \end{cases} ;t\in[0,1]$$

Proposed by Daniel Sitaru- Romania

UP.619. Find the curvilinear integral of the second kind:

$$\int_C yzdx+xzdy+xydz, \qquad C: \begin{cases} x(t)=t \\ y(t)=2t \\ z(t)=3t \end{cases} ;t\in[0,1]$$

Proposed by Daniel Sitaru - Romania

UP.620. Find the curvilinear integral of the second kind:

$$\Omega=\int_\gamma (x+2y+z)ds; \qquad \gamma=[AB];A(1,2,3);B(3,4,5)$$

Proposed by Daniel Sitaru - Romania

UP.621. Find the curvilinear integral of the kind kind:

$$\Omega=\int_\gamma (x+2y+z)ds; \qquad \gamma=[AB];A(1,2,3);B(3,4,5)$$

Proposed by Daniel Sitaru - Romania

UP.622. Find the surface integral of the first kind:

$$\Omega=\int\int_{\sum}(x+y+z)d\tau$$

$$\sum=\{(x,y,z)\in\mathbb{R}^3|x=2u+3v,y=3u+2z,z=u+v;u,v\in[0,1]\}$$

Proposed by Daniel Sitaru - Romania

UP.623. If $a,b,c>0,a^2+b^2+c^2=1$ and $\lambda\ge\frac{1}{3}$ then:

$$\sum a^2b\sum\frac{1}{\sqrt{a^2+\lambda}}\le\frac{3}{\sqrt{1+3\lambda}}$$

Proposed by Marin Chirciu - Romania

UP.624. Find:

$$\Omega=\int\int_D (2x+3y)dxdy$$

$$D=Int(\Delta AOB);O(0,0);A(2,3);B(2,5)$$

Proposed by Daniel Sitaru - Romania

UP.625. Find:

$$\Omega = \int\int_D (3x+2y)dxdy$$

$$D = Int(\Delta AOB); O(0,0); A(2,4); B(5,4)$$

Proposed by Daniel Sitaru - Romania

UP.626. Find:

$$\Omega = \int\int_D (2x+3y)dxdy$$

$$D = Int(\Delta AOB); O(0,0); A(2,3); B(2,5)$$

Proposed by Daniel Sitaru - Romania

UP.627. Find the surface integral of the second kind:

$$\int\int_{\sum} 2xdydz + ydzdx + 5zdxdy$$

where $\sum$ is the surface of the ellipsoid:

$$E : \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{25} = 1$$

Proposed by Daniel Sitaru - Romania

UP.628. Find the general solution for the system:

$$\begin{cases} y_1' = y_1 + y_2 + y_3 \\ y_2' = 2y_2 + y_3 \\ y_3' = 3y_3 \end{cases}$$

Proposed by Daniel Sitaru - Romania

UP.629. Find the general solution:

$$y'' - 5y' + 4y = xe^x$$

Proposed by Daniel Sitaru - Romania

UP.630. Find:

$$\Omega = \int\int_D x^2 dxdy$$

$$D = \{(x,y)|0 \le y \le 9 - x^2; -3 \le x \le 3\}$$

Proposed by Daniel Sitaru - Romania

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