

ROMANIAN MATHEMATICAL MAGAZINE

Solve for $x \in \left(0, \frac{\pi}{2}\right)$:

$$\tan x + 4\sin x = \sqrt{3}$$

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$$\tan x + 4\sin x = \sqrt{3}$$

$$\frac{\sin x}{\cos x} + 4\sin x = \sqrt{3}$$

$$\sin x + 4\sin x \cos x = \sqrt{3} \cos x$$

$$\sin x - \sqrt{3} \cos x + 2(2\sin x \cos x) = 0$$

$$\frac{1}{2} \sin x - \frac{\sqrt{3}}{2} \cos x + \sin(2x) = 0,$$

$$\sin x \cos \frac{\pi}{3} - \cos x \sin \frac{\pi}{3} + \sin(2x) = 0$$

$$\sin\left(x - \frac{\pi}{3}\right) + \sin(2x) = 0$$

$$\sin(2x) = \sin\left(\frac{\pi}{3} - x\right)$$

$$2x = \frac{\pi}{3} - x + 2k\pi \quad \vee \quad 2x = \pi - \left(\frac{\pi}{3} - x\right) + 2k\pi, \quad k \in \mathbb{Z}$$

$$3x = \frac{\pi}{3} + 2k\pi \quad \vee \quad 2x = \frac{2\pi}{3} + x + 2k\pi$$

$$x = \frac{\pi}{9} + \frac{2k\pi}{3} \quad \vee \quad x = \frac{2\pi}{3} + 2k\pi$$

$$x \in \left(0, \frac{\pi}{2}\right) \Rightarrow k = 0$$

$$x = \frac{\pi}{9} \quad \vee \quad x = \frac{2\pi}{3} \notin \left(0, \frac{\pi}{2}\right)$$

$$\therefore x = \frac{\pi}{9}$$