ROMANIAN MATHEMATICAL MAGAZINE

SP.615 Let be $A(1,2,3)$ and the real plan: $P: 2x+y+z-5=0$

Find the parametrical equations of the perpendicular line from A to the real plan P.

Proposed by Daniel Sitaru – Romania

Solution 1 by proposer

$P: 2x+y+z-5=0$. The normal vector of P is $\vec{u}(2,1,1)$.

$$\vec{u}=2\vec{i}+\vec{j}+\vec{k}$$

$$d: \frac{x-x_A}{2}=\frac{y-y_A}{1}=\frac{z-z_A}{1}$$

$$d: \frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{1}=t; t\in\mathbb{R}$$

$$x-1=2t; y-2=t; z-3=t$$

$$d: \begin{cases} x=1+2t \\ y=2+t \\ z=3+t \end{cases}; t\in\mathbb{R}$$

Solution 2 by José Luis Díaz – Barrero – Spain

To find the parametric equations of the line that is perpendicular to the plane P and passes through the point $A(1,2,3)$, we follow these steps:

1. Find the direction vector of the line. The equation of the given plane is:

$$P: 2x+y+z-5=0$$

The coefficients of x, y and z in the plane's equation give us its normal vector $\vec{n}$, which is perpendicular to the plane $\vec{n}=(2,1,1)$. Since the line we want to find is perpendicular to the plane, its direction vector $\vec{v}$ must be parallel to the plane's normal vector. Therefore, we can use the normal vector as the direct ion vector for our line $\vec{v}=(2,1,1)$.

2. Write the parametric equations. The parametric equations of a line passing through a point $A(x_0,y_0,z_0)$ with a direction vector $\vec{v}=(a,b,c)$ are given by:

$$x=x_0+at$$

$$y=y_0+bt$$

$$z=z_0+ct$$

where t is a real parameter $t \in \mathbb{R}$. Substituting the coordinates of the point $A(1,2,3)$ and the components of the direction vector $\vec{v} = (2,1,1)$, we get:

$$x = 1 + 2t$$

$$y = 2 + 1t$$

$$z = 3 + 1t$$

Thus, the parametric equations of the perpendicular line are:

$$\begin{cases} x = 1 + 2t \\ y = 2 + t \\ z = 3 + t \end{cases}$$

where $t \in \mathbb{R}$.