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SP.615. *Proposed by Daniel Sitaru, Romania.* Given the point $A(1,2,3)$ and the plane $P: 2x+y+z-5=0$ in $\mathbb{R}^3$, find the parametric equations of the line passing through A perpendicular to P.

Solution by José Luis Díaz-Barrero, Barcelona, Spain. To find the parametric equations of the line that is perpendicular to the plane P and passes through the point $A(1,2,3)$, we follow these steps:

1. Find the direction vector of the line. The equation of the given plane is:

$$P: 2x+y+z-5=0$$

The coefficients of x, y, and z in the plane's equation give us its normal vector $\vec{n}$, which is perpendicular to the plane $\vec{n}=(2,1,1)$. Since the line we want to find is perpendicular to the plane, its direction vector $\vec{v}$ must be parallel to the plane's normal vector. Therefore, we can use the normal vector as the direct ion vector for our line $\vec{v}=(2,1,1)$.

2. Write the parametric equations. The parametric equations of a line passing through a point $A(x_0,y_0,z_0)$ with a direction vector $\vec{v}=(a,b,c)$ are given by:

$$\begin{aligned} x &= x_0 + at \\ y &= y_0 + bt \\ z &= z_0 + ct \end{aligned}$$

where t is a real parameter $t \in \mathbb{R}$. Substituting the coordinates of the point $A(1,2,3)$ and the components of the direction vector $\vec{v}=(2,1,1)$, we get:

$$\begin{aligned} x &= 1 + 2t \\ y &= 2 + 1t \\ z &= 3 + 1t \end{aligned}$$

Thus, the parametric equations of the perpendicular line are:

$$\begin{cases} x = 1 + 2t \\ y = 2 + t \\ z = 3 + t \end{cases}$$

where $t \in \mathbb{R}$.