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SP.614. *Proposed by Daniel Sitaru, Romania.* Given the points $A(1,3,2)$, $B(3,1,1)$, and $C(4,2,0)$, find the area of the parallelogram built on the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

Solution by José Luis Díaz-Barrero, Barcelona, Spain. The area of a parallelogram spanned by two vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is equal to the magnitude (norm) of their cross product:

$$\text{Area} = \|\overrightarrow{AB} \times \overrightarrow{AC}\|$$

Now, we find the components of the vectors

$$\overrightarrow{AB} = (3-1, 1-3, 1-2) = (2, -2, -1)$$

$$\overrightarrow{AC} = (4-1, 2-3, 0-2) = (3, -1, -2)$$

The cross product is given by

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -2 & -1 \\ 3 & -1 & -2 \end{vmatrix} = (3, 1, 4)$$

The area of the parallelogram S is the magnitude of this vector. That is,

$$S = \|\overrightarrow{AB} \times \overrightarrow{AC}\| = \sqrt{3^2 + 1^2 + 4^2} = \sqrt{26}$$