ROMANIAN MATHEMATICAL MAGAZINE

SP.613 Let be $A(2,4,1); B(4,2,5)$. Find the mediator plan of the segment $[AB]$.

Proposed by Daniel Sitaru – Romania

Solution 1 by proposer

$$M\left(\frac{x_A+x_B}{2},\frac{y_A+y_B}{2},\frac{z_A+z_B}{2}\right)\Rightarrow M\left(\frac{2+4}{2};\frac{4+2}{2};\frac{1+5}{2}\right)$$

$$\Rightarrow M(3,3,3)$$

$$\overrightarrow{AB}=(x_B-x_A)\vec{i}+(y_B-y_A)\vec{j}+(z_B-z_A)\vec{k}$$

$$\overrightarrow{AB}=(4-2)\vec{i}+(2-4)\vec{j}+(5-1)\vec{k}$$

$$\overrightarrow{AB}=2\vec{i}-2\vec{j}+4\vec{k}$$

Let P be the mediator plan of the segment $[AB]$.

$$P:(x-x_M)\cdot 2+(y-y_M)\cdot(-2)+(z-z_M)\cdot 4=0$$

$$P:(x-3)\cdot 2+(y-3)\cdot(-2)+(z-3)\cdot 4=0$$

$$P:2x-6-2y+6+4z-12=0$$

$$P:2x-2y+4z-12=0$$

$$P:x-y+2z-6=0$$

Solution 2 by Marin Chirciu-Romania

M the middle of the segment $AB\Rightarrow$

$$M(x_M,y_M,z_M),x_M=\frac{x_A+x_B}{2},y_M=\frac{y_A+y_B}{2},z_M=\frac{z_A+z_B}{2}\Rightarrow$$

$$\Rightarrow x_M=\frac{2+4}{2},y_M=\frac{4+2}{2},z_M=\frac{1+5}{2}\Rightarrow x_M=3,y_M=3,z_M=3\Rightarrow M(3,3,3).$$

$\overrightarrow{AB}$ is the normal vector of the mediator plan of $[AB]$

$$(x_B-x_A)(x-x_M)+(y_B-y_A)(y-y_M)+(z_B-z_A)(z-z_M)=0\Leftrightarrow$$

$$\Leftrightarrow(4-2)(x-3)+(2-4)(y-3)+(5-1)(z-3)=0\Leftrightarrow 2(x-3)-2(y-3)+4(z-3)=0\Leftrightarrow$$

$$\Leftrightarrow 2(x-3)-2(y-3)+4(z-3)=0\Leftrightarrow x-y+2z-6=0$$

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Solution 3 by Jose Luis Diaz Barrero-Spain

To find the mediator plane (the perpendicular bisector plane) of the segment $[AB]$ given the points $A(2,4,1)$ and $B(4,2,5)$, we need two elements:

1. A point on the plane, which is the midpoint M of the segment $[AB]$.
2. A normal vector $\vec{n}$ to the plane, which is the vector $\overrightarrow{AB}$.

The coordinates of the midpoint M are calculated by taking the average of the coordinates of A and B:

$$M = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}, \frac{z_A + z_B}{2}\right)$$

$$M = \left(\frac{2+4}{2}, \frac{4+2}{2}, \frac{1+5}{2}\right) = (3,3,3)$$

The normal vector $\vec{n}$ is directed along the segment AB:

$$\vec{n} = \overrightarrow{AB} = (x_B - x_A, y_B - y_A, z_B - z_A)$$

$$\vec{n} = (4-2, 2-4, 5-1) = (2,-2,4)$$

To simplify the equation of the plane, we cand divide the vector components by their greatest common divisor, 2, to obtain a simpler normal vector:

$$\vec{n} = (1,-1,2)$$

The standard equation of a plane with a normal vector $\vec{n} = (a,b,c)$ passing through a point $M(x_0, y_0, z_0)$ is:

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Substituting $\vec{n} = (1,-1,2)$ and $M(3,3,3)$ into the formula, yields the Cartesian equation of the mediator plane of the segment $[AB]$:

$$x - y + 2z - 6 = 0$$