José Luis **Díaz-Barrero**
Barcelona Mathematical Circle (BMC)
jose.luis.diaz@upc.edu

SP.613. *Proposed by Daniel Sitaru, Romania.* Let $A(2,4,1), B(4,2,5)$ b two points in $\mathbb{R}^3$. Find the mediator plan of the segment $[AB]$.

Solution by José Luis Díaz-Barrero, Barcelona, Spain. To find the mediator plane (the perpendicular bisector plane) of the segment $[AB]$ given the points $A(2,4,1)$ and $B(4,2,5)$, we need two elements:

1. A point on the plane, which is the midpoint M of the segment $[AB]$.
2. A normal vector $\vec{n}$ to the plane, which is the vector $\overrightarrow{AB}$.

The coordinates of the midpoint M are calculated by taking the average of the coordinates of A and B:

$$M = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}, \frac{z_A + z_B}{2}\right)$$

$$M = \left(\frac{2+4}{2}, \frac{4+2}{2}, \frac{1+5}{2}\right) = (3,3,3)$$

The normal vector $\vec{n}$ is directed along the segment AB:

$$\vec{n} = \overrightarrow{AB} = (x_B - x_A,\, y_B - y_A,\, z_B - z_A)$$

$$\vec{n} = (4-2,\, 2-4,\, 5-1) = (2,-2,4)$$

To simplify the equation of the plane, we can divide the vector components by their greatest common divisor, 2, to obtain a simpler normal vector:

$$\vec{n} = (1,-1,2)$$

The standard equation of a plane with a normal vector $\vec{n} = (a,b,c)$ passing through a point $M(x_0, y_0, z_0)$ is:

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Substituting $\vec{n} = (1,-1,2)$ and $M(3,3,3)$ into the formula, yields the Cartesian equation of the mediator plane of the segment $[AB]$:

$$x - y + 2z - 6 = 0$$