ROMANIAN MATHEMATICAL MAGAZINE

SP.613 Let be $A(2,4,1)$; $B(4,2,5)$. Find the mediator plan of the segment $[AB]$.

Proposed by Daniel Sitaru – Romania

Solution 1 by proposer

$$M\left(\frac{x_A+x_B}{2},\frac{y_A+y_B}{2},\frac{z_A+z_B}{2}\right)\Rightarrow M\left(\frac{2+4}{2};\frac{4+2}{2};\frac{1+5}{2}\right)$$

$$\Rightarrow M(3,3,3)$$

$$\overrightarrow{AB}=(x_B-x_A)\vec{\imath}+(y_B-y_A)\vec{\jmath}+(z_B-z_A)\vec{k}$$

$$\overrightarrow{AB}=(4-2)\vec{\imath}+(2-4)\vec{\jmath}+(5-1)\vec{k}$$

$$\overrightarrow{AB}=2\vec{\imath}-2\vec{\jmath}+4\vec{k}$$

Let P be the mediator plan of the segment $[AB]$.

$$P:(x-x_M)\cdot 1+(y-y_M)\cdot(-2)+(z-z_M)\cdot 4=0$$

$$P:(x-3)\cdot 1+(y-3)\cdot(-2)+(z-3)\cdot 4=0$$

$$P:x-3-2y+6+4z-12=0$$

$$P:x-2y+4z-6=0$$

Solution 2 by Marin Chirciu-Romania

M the middle of the segment $AB\Rightarrow$

$$M(x_M,y_M,z_M),\ x_M=\frac{x_A+x_B}{2},\ y_M=\frac{y_A+y_B}{2},\ z_M=\frac{z_A+z_B}{2}\Rightarrow$$

$$\Rightarrow x_M=\frac{2+4}{2},\ y_M=\frac{4+2}{2},\ z_M=\frac{1+5}{2}\Rightarrow x_M=3,y_M=3,z_M=3\Rightarrow M(3,3,3).$$

$\overrightarrow{AB}$ is the normal vector of the mediator plan of $[AB]$

$$(x_B-x_A)(x-x_M)+(y_B-y_A)(y-y_M)+(z_B-z_A)(z-z_M)=0\Leftrightarrow$$

$$\Leftrightarrow(4-2)(x-3)+(2-4)(y-3)+(5-1)(z-3)=0\Leftrightarrow 2(x-3)-2(y-3)+4(z-3)=0\Leftrightarrow$$

$$\Leftrightarrow 2(x-3)-2(y-3)+4(z-3)=0\Leftrightarrow x-y+2z-6=0.$$