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SP.612. *Proposed by Daniel Sitaru, Romania.* Let $A(1,2,3)$, $B(4,1,1)$, $C(0,3,2)$, and $D(2,1,3)$ be four points in $\mathbb{R}^3$. Find the volume of the parallelepiped builded on the vectors $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$.

Solution by José Luis Díaz-Barrero, Barcelona, Spain. To find the volume V of the parallelepiped built on the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$, and $\overrightarrow{AD}$ from the points $A(1,2,3)$, $B(4,1,1)$, $C(0,3,2)$, and $D(2,1,3)$, we use the scalar triple product. The volume is given by the absolute value of the determinant of the matrix formed by these three vectors:

$$V = \left| \vec{AB} \cdot (\vec{AC} \times \vec{AD}) \right|$$

We find each vector by subtracting the coordinates of the initial point from the terminal point:

$$\begin{aligned}
\overrightarrow{AB} &= (4-1,\, 1-2,\, 1-3) = (3,-1,-2) \\
\overrightarrow{AC} &= (0-1,\, 3-2,\, 2-3) = (-1,1,-1) \\
\overrightarrow{AD} &= (2-1,\, 1-2,\, 3-3) = (1,-1,0)
\end{aligned}$$

The volume is the absolute value of the determinant of the 3×3 matrix containing these vectors as rows:

$$V = \left| \det \begin{pmatrix} 3 & -1 & -2 \\ -1 & 1 & -1 \\ 1 & -1 & 0 \end{pmatrix} \right| = |-2| = 2$$