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MOTION PROBLEMS

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Motion problems (specifically, uniform motion) constitute a distinct topic in mathematics at the transition from primary to lower secondary school; in these problems, speed (v) is defined as the ratio of displacement (or distance traveled, d) to the time (t) required for the motion: $v = \frac{d}{t}$. This material briefly outlines several types of motion problems and then presents solutions to arithmetic problems—which might otherwise fall into different categories—by applying the concept of this "speed" relationship. In the case of kinematics problems (from the Greek “kinetikos”, meaning "motion"), a particular distinction is made regarding problems involving the meeting of moving bodies.

1. Problems of meeting of mobiles, when they move in the same direction

In this case (see fig. 1), the meeting time (t_{I_1}) of the mobiles starting from points A and B, with speeds v_1 respectively v_2 can be deduced as follows:

$$AB + BI = AI \qquad (1)$$

where I = the meeting point of the two mobiles.

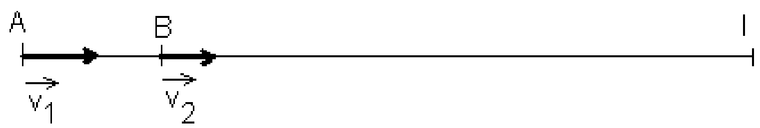

Fig. 1

Taking into account the relationship between speed, time and distance, the relationship (1) can be rewritten as follows: $AB + v_2 \cdot t_{I_1} = v_1 \cdot t_{I_1}$

from which the meeting time results:

$$t_{I\,(1)} = \frac{AB}{v_1 - v_2} \qquad (2)$$

The distances covered by the two mobiles until the moment of meeting are:

$$AI_{(1)} = v_1 \cdot t_{I(1)} = v_1 \cdot \frac{AB}{v_1 - v_2}, \qquad (3)$$

$$BI_{(1)} = v_2 \cdot t_{I(1)} = v_2 \cdot \frac{AB}{v_1 - v_2}$$

2. Problems of meeting of mobiles, when they move in opposite directions

In this case, the meeting time (t_{I_2}) of the two mobiles will be deduced (see fig. 2), from the relation: $AI + IB = AB$ equivalent to: $v_1 \cdot t_I + v_2 \cdot t_I = AB$ where:

$$t_{I\,(2)} = \frac{AB}{v_1 + v_2} \qquad (4)$$

The distances traveled by the two mobiles until the moment of meeting will be:

$$AI_{(2)} = v_1 \cdot t_{I(2)} = v_1 \cdot \frac{AB}{v_1 + v_2} \qquad (5)$$

$$BI_{(2)} = v_2 \cdot t_{I(2)} = v_2 \cdot \frac{AB}{v_1+v_2} \qquad (6)$$

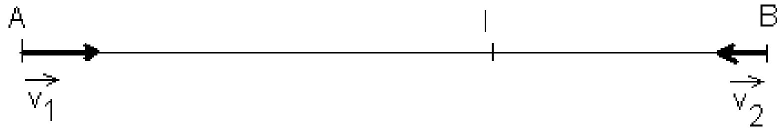

Fig. 2

3. Problems that can be solved by extending the notion of speed and addressing cinematic techniques

Problem 1: An employee of a publishing house can proofread a manuscript in 8 hours, and another in 12 hours. In how long would they finish proofreading the manuscript, if they worked together?

***Solution 1* (Reduction to unity method):**

First copywriter ………………….1 hour ………………………..1/8 of the paper

Second copywriter …………….. 1 hour ………………………..1/12 of the paper

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Together the two copywriters …..1 hour …….(1/8+1/12) of the paper=5/24 of the paper

Together the two copywriters ….24/5 hours=4.8 hours=288 minutes…. 1 paper

***Solution 2* ("Kinematic" method):**

We define the (constant) working speed v of a publishing house employee by the number (n) of pages typed per unit of time (t): $v = \frac{n}{t}$ (noting that the place of the "distance" d covered in the motion problems is now taken by the "number of pages typed" n).

If we denote by N the number of pages of the manuscript and by t_x the time, expressed in hours, necessary for the two typewriters to finish the manuscript if they worked together, then the number of pages of the manuscript is equal to the sum of the number of pages typed by each of the two employees: $n_1 + n_2 = N$

Substituting into the above equality, from the definition relation of the working speed:

$n_1 = v_1 \cdot t_x = \frac{N}{8} \cdot t_x$, $n_2 = v_2 \cdot t_x = \frac{N}{12} \cdot t_x$, we get: $v_1 \cdot t_x + v_2 \cdot t_x = N$

$$t_x = \frac{N}{v_1+v_2}$$

that is, a relationship similar to relationship (4), from the case of meeting problems when the mobiles moved in opposite directions, obviously changing the "distance" with the "number of pages".

$$t_x = \frac{N}{v_1+v_2} = \frac{N}{\frac{N}{8}+\frac{N}{12}} = \frac{1}{\frac{5}{24}} = \frac{24}{5}\ hours = 4{,}8\ hours = 288\ minutes$$

Problem 2: If I solve 15 exercises per day during my vacation, I have 3 days left free, and if I solve 12 exercises per day, then 18 problems remain unsolved. How many exercises and how many vacation days do I have?

Solution: We define the speed (v) of solving as the ratio between the number (n) of exercises solved and the time (t) of solving: $v=\frac{n}{t}$ and we have for the two situations the "speeds": v_1=15 exercises solved/day, v_2=12 exercises solved/day.

The total number (N) of exercises proposed for solving, during time T (vacation days) will be: $v_1\cdot(T-3)=v_2\cdot T+18$, $v_1\cdot T-v_2\cdot T=3\cdot v_1+18$ where do we get the number of vacation days:

$$T=\frac{3\cdot v_1+18}{v_1-v_2}=\frac{3\cdot 15+18}{15-12}=\frac{63}{3}=21\ days$$

The total number of exercises to be solved is:

$N=v_1\cdot(T-3)=15\cdot(21-3)=270\ exercises.$

Problem 3: A pool can be filled through two taps in 12 hours. The first tap can fill the pool in 10 hours less than the filling time of the second. In how long can each tap, separately, fill the pool?

Solution: We can quantify the "speed" of water flow through each tap, for example, by the volume of water drained per unit of time. Denoting V = the volume of the pool, T1, T2 = the times of filling the pool by each of the two taps, we have: $v_1=\frac{V}{T_1},\ v_2=\frac{V}{T_2}$

Writing the volume of water in the pool as the sum of the volumes of water flowing through the two taps for 12 hours and the difference between the filling times of 10 hours, we obtain the system of equations:

$$\begin{cases} v_1\cdot 12+v_2\cdot 12=V \\ T_1-T_2=10 \end{cases} \leftrightarrow \begin{cases} \frac{V}{T_1}\cdot 12+\frac{V}{T_2}\cdot 12=V \\ T_1-T_2=10 \end{cases} \leftrightarrow \begin{cases} \frac{1}{T_1}+\frac{1}{T_2}=\frac{1}{12} \\ T_1-T_2=10 \end{cases}$$

$$\begin{cases} T_1=T_2+10 \\ \frac{1}{T_2+10}+\frac{1}{T_2}=\frac{1}{12} \end{cases} \leftrightarrow \begin{cases} T_1=T_2+10 \\ {T_2}^2-14T_2-120=0 \end{cases} \leftrightarrow \begin{cases} T_2=\frac{14+26}{2}=20\ hours \\ T_1=20+10=30\ hours \end{cases}$$

The problems solved, even if they do not contain specific quantities of movement (speed, distance covered), imply a certain "rhythm" of the action and the "time" factor, which allows them such an approach.

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