ROMANIAN MATHEMATICAL MAGAZINE

JP.613 Find the angle between the lines:

$$d_1:\begin{cases} x = 3 - 2t \\ y = 1 + t \\ z = -1 + 3t \end{cases};\ d_2:\begin{cases} x = 2 + 4t \\ y = -1 + 2t \\ z = 4 - t \end{cases};\ t \in \mathbb{R}$$

Proposed by Daniel Sitaru – Romania

Solution 1 by proposer

$$d_1:\begin{cases} \frac{x-3}{-2} = t \\ \frac{y-1}{1} = t \\ \frac{z+1}{3} = t \end{cases};\ d_2:\begin{cases} \frac{x-2}{4} = t \\ \frac{y+1}{2} = t \\ \frac{z-4}{-1} = t \end{cases};\ t \in \mathbb{R}$$

$$d_1: \frac{x-3}{-2} = \frac{y-1}{1} = \frac{z+1}{3};\ d_2: \frac{x-2}{4} = \frac{y+1}{2} = \frac{z-4}{-1}$$

The directions vectors of d_1, d_2:

$$\overrightarrow{u_1}(-2,1,3) = -2\vec{i} + \vec{j} + 3\vec{k}$$

$$\overrightarrow{u_2}(4,2,-1) = 4\vec{i} + 2\vec{j} - \vec{k}$$

$$\overrightarrow{u_1} \cdot \overrightarrow{u_2} = -2 \cdot 4 + 1 \cdot 2 + 3 \cdot (-1) = -8 + 2 - 3 = -9$$

$$|\overrightarrow{u_1}| = \sqrt{(-2)^2 + 1^2 + 3^2} = \sqrt{4 + 1 + 9} = \sqrt{14}$$

$$|\overrightarrow{u_2}| = \sqrt{4^2 + 2^2 + (-1)^2} = \sqrt{16 + 4 + 1} = \sqrt{21}$$

$$\cos\left(\sphericalangle(\overrightarrow{u_1}, \overrightarrow{u_2})\right) = \frac{\overrightarrow{u_1} \cdot \overrightarrow{u_2}}{|\overrightarrow{u_1}| \cdot |\overrightarrow{u_2}|} = \frac{-9}{\sqrt{14} \cdot \sqrt{21}} = \frac{-9}{7\sqrt{6}} = \frac{-9\sqrt{6}}{7 \cdot 6} = \frac{-3\sqrt{6}}{14}$$

$$\mu\left(\sphericalangle(\overrightarrow{u_1}, \overrightarrow{u_2})\right) = \arccos\left(\frac{-3\sqrt{6}}{14}\right) = \pi - \arccos\left(\frac{3\sqrt{6}}{14}\right)$$

Solution 2 by José Luis Díaz – Barrero – Spain

To find the acute angle θ between the two lines d_1 and d_2, we need to find the angle between their respective direction vectors. These vectors are formed by the coefficients of the parameter t, as it is well-known.

For the first line d_1, the direction vector is $\overrightarrow{v_1} = (-2, 1, 3)$.

For the second line d_2, the direction vector is $\overrightarrow{v_2} = (4, 2, -1)$.

The acute angle θ between two lines with direction vectors $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$ is given by:

$$\cos(\theta) = \frac{|\overrightarrow{v_1} \cdot \overrightarrow{v_2}|}{||\overrightarrow{v_1}||\,||\overrightarrow{v_2}||}$$

1. Dot Product ($\overrightarrow{v_1} \cdot \overrightarrow{v_2}$):

$$\overrightarrow{v_1} \cdot \overrightarrow{v_2} = (-2)(4) + (1)(2) + (3)(-1) = -8 + 2 - 3 = -9$$

Taking the absolute value yields: $|\overrightarrow{v_1} \cdot \overrightarrow{v_2}| = 9$.

2. Magnitude of $\overrightarrow{v_1}$:

$$||\overrightarrow{v_1}|| = \sqrt{(-2)^2 + 1^2 + 3^2} = \sqrt{4 + 1 + 9} = \sqrt{14}$$

3. Magnitude of $\overrightarrow{v_2}$:

$$||\overrightarrow{v_2}|| = \sqrt{4^2 + 2^2 + (-1)^2} = \sqrt{16 + 4 + 1} = \sqrt{21}$$

4. Product of Magnitudes:

$$||\overrightarrow{v_1}||\,||\overrightarrow{v_2}|| = \sqrt{14} \cdot \sqrt{21} = \sqrt{294} = 7\sqrt{6}$$

Substituting the calculated values into the formula, yields

$$\cos(\theta) = \frac{9}{7\sqrt{6}} = \frac{3\sqrt{6}}{14}$$

Finally, taking the inverse cosine, we obtain

$$\theta = \arccos\left(\frac{3\sqrt{6}}{14}\right)$$