ROMANIAN MATHEMATICAL MAGAZINE

JP.613 Find the angle between the lines:

$$d_1:\begin{cases} x=3-2t \\ y=1+t \\ z=-1+3t \end{cases};d_2:\begin{cases} x=2+4t \\ y=-1+2t \\ z=4-t \end{cases};t\in\mathbb{R}$$

Proposed by Daniel Sitaru – Romania

Solution 1 by proposer

$$d_1:\begin{cases} \frac{x-3}{-2}=t \\ \frac{y-1}{1}=t \\ \frac{z+1}{3}=t \end{cases};d_2:\begin{cases} \frac{x-2}{4}=t \\ \frac{y+1}{2}=t \\ \frac{z-4}{-1}=t \end{cases};t\in\mathbb{R}$$

$$d_1:\frac{x-3}{-2}=\frac{y-1}{1}=\frac{z+1}{3};d_2:\frac{x-2}{4}=\frac{y+1}{2}=\frac{z-4}{-1}$$

The directions vectors of d_1,d_2:

$$\overrightarrow{u_1}(-2,1,3)=-2\vec{i}+\vec{j}+3\vec{k}$$

$$\overrightarrow{u_2}(4,2,-1)=4\vec{i}+2\vec{j}-\vec{k}$$

$$\overrightarrow{u_1}\cdot\overrightarrow{u_2}=-2\cdot4+1\cdot2+3\cdot(-1)=-8+2-3=-9$$

$$|\overrightarrow{u_1}|=\sqrt{(-2)^2+1^2+3^2}=\sqrt{4+1+9}=\sqrt{14}$$

$$|\overrightarrow{u_2}|=\sqrt{4^2+2^2+(-1)^2}=\sqrt{16+4+1}=\sqrt{21}$$

$$\cos\left(\sphericalangle(\overrightarrow{u_1},\overrightarrow{u_2})\right)=\frac{\overrightarrow{u_1}\cdot\overrightarrow{u_2}}{|\overrightarrow{u_1}|\cdot|\overrightarrow{u_2}|}=\frac{-9}{\sqrt{14}\cdot\sqrt{21}}=\frac{-9}{7\sqrt{6}}=\frac{-9\sqrt{6}}{7\cdot6}=\frac{-3\sqrt{6}}{14}$$

$$\mu\left(\sphericalangle(\overrightarrow{u_1},\overrightarrow{u_2})\right)=\arccos\left(\frac{-3\sqrt{6}}{14}\right)=\pi-\arccos\left(\frac{3\sqrt{6}}{14}\right)$$

Solution 2 by José Luis Díaz – Barrero – Spain

To find the acute angle θ between the two lines d_1 and d_2, we need to find the angle between their respective direction vectors. These vectors are formed by the coefficients of the parameter t, as it is well-known.

For the first line d_1, the direction vector is $\overrightarrow{v_1}=(-2,1,3)$.

For the second line d_2, the direction vector is $\overrightarrow{v_2}=(4,2,-1)$.

The acute angle θ between two lines with direction vectors $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$ is given by:

$$\cos(\theta) = \frac{|\overrightarrow{v_1} \cdot \overrightarrow{v_2}|}{||\overrightarrow{v_1}||||\overrightarrow{v_2}||}$$

1. Dot Product $(\overrightarrow{v_1} \cdot \overrightarrow{v_2})$:

$$\overrightarrow{v_1} \cdot \overrightarrow{v_2} = (-2)(4) + (1)(2) + (3)(-1) = -8 + 2 - 3 = -9$$

Taking the absolute value yields: $|\overrightarrow{v_1} \cdot \overrightarrow{v_2}| = 9$.

2. Magnitude of $\overrightarrow{v_1}$:

$$||\overrightarrow{v_1}|| = \sqrt{(-2)^2 + 1^2 + 3^2} = \sqrt{4 + 1 + 9} = \sqrt{14}$$

3. Magnitude of $\overrightarrow{v_2}$:

$$||\overrightarrow{v_2}|| = \sqrt{4^2 + 2^2 + (-1)^2} = \sqrt{16 + 4 + 1} = \sqrt{21}$$

4. Product of Magnitudes:

$$||\overrightarrow{v_1}||||\overrightarrow{v_2}|| = \sqrt{14} \cdot \sqrt{21} = \sqrt{294} = 7\sqrt{6}$$

Substituting the calculated values into the formula, yields

$$\cos(\theta) = \frac{9}{7\sqrt{6}} = \frac{3\sqrt{6}}{14}$$

Finally, taking the inverse cosine, we obtain

$$\theta = \arccos\left(\frac{3\sqrt{6}}{14}\right)$$

Solution 3 by Omkumar Rao-India

The angle between two lines is determined by the angle between their direction vectors. For line d_1, the direction vector is $\vec{v}_1 = (-2, 1, 3)$, and for line d_2, the direction vector: $\vec{v}_2 = (4, 2, -1)$

The angle θ between the lines is given by:

$$\cos\theta = \frac{|\vec{v}_1 . \vec{v}_2|}{|\vec{v}_1| . |\vec{v}_2|}$$

$$\vec{v}_1 . \vec{v}_2 = -9$$

$$|\vec{v}_1| = \sqrt{14}\ , |\vec{v}_2| = \sqrt{21}$$

$$\cos\theta = \frac{3\sqrt{6}}{14}$$

The angle between the lines d_1 and d_2 is

$$\theta = \cos^{-1}\left(\frac{3\sqrt{6}}{14}\right)$$