ROMANIAN MATHEMATICAL MAGAZINE

JP.612 Find the angle between the line d and the real plan P.

$$d:\begin{cases} x = 2+3t \\ y = 3-2t \\ z = 1+5t \end{cases} ; t\in\mathbb{R}; P: 2x+y+z-5=0$$

Proposed by Daniel Sitaru – Romania

Solution 1 by proposer

$$d:\begin{cases} x-2 = 3t \\ y-3 = -2t \\ z-1 = 5t \end{cases} \Rightarrow \begin{cases} \frac{x-2}{3} = t \\ \frac{y-3}{-2} = t \\ \frac{z-1}{5} = t \end{cases} ; t\in\mathbb{R}$$

$$d: \frac{x-2}{3} = \frac{y-3}{-2} = \frac{z-1}{5}$$

The directory vector of d is:

$$\vec{v}(3,-2,5) = 3\vec{i} - 2\vec{j} + 5\vec{k}$$

The normal vector of P is:

$$\vec{u}(2,1,1) = 2\vec{i} + \vec{j} + \vec{k}$$

$$\vec{u}\cdot\vec{v} = 3\cdot 2 - 2\cdot 1 + 5\cdot 1 = 6 - 2 + 5 = 9$$

$$|\vec{u}| = \sqrt{2^2+1^2+1^2} = \sqrt{6}$$

$$|\vec{v}| = \sqrt{3^2+(-2)^2+5^2} = \sqrt{9+5+25} = \sqrt{38}$$

$$\cos(\sphericalangle(\vec{u},\vec{v})) = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|} = \frac{9}{\sqrt{6}\cdot\sqrt{38}} = \frac{9}{2\sqrt{3\cdot 19}} = \frac{9\sqrt{57}}{2\cdot 3\cdot 19}$$

$$\cos(\sphericalangle(\vec{u},\vec{v})) = \frac{3\sqrt{57}}{38}$$

$$\mu(\sphericalangle(\vec{u},\vec{v})) = \arccos\left(\frac{3\sqrt{57}}{38}\right)$$

$$\sphericalangle(d,P) = \frac{\pi}{2} - \arccos\left(\frac{3\sqrt{57}}{38}\right) = arcsin\left(\frac{3\sqrt{57}}{38}\right)$$

Solution 2 by José Luis Díaz – Barrero – Spain

To find the angle θ between a line and a plane, we use the relationship between the direction vector of the line $\vec{u}$ and the normal vector of the plane $\vec{n}$. Because the normal vector is perpendicular to the plane, the angle α between $\vec{u}$ and $\vec{n}$ is complementary to the angle θ we want to find $\theta = 90^\circ - \alpha$. Therefore, we use the sine formula:

$$\sin(\theta) = \frac{|\vec{u}\cdot\vec{n}|}{||\vec{u}|||\vec{n}||}$$

- **Direction vector of the line $\vec{u}$: Look at the coefficients of the parameter t in the line's parametric equations, to obtain**

$$\vec{u} = (3, -2, 5)$$

- **Normal vector of the plane $\vec{n}$: Look at the coefficients of x, y, and z in the plane's equation, to obtain**

$$\vec{n} = (2, 1, 1)$$

Then,

$$\vec{u}\cdot\vec{n} = (3)(2) + (-2)(1) + (5)(1) = 9,$$

and $|\vec{u}\cdot\vec{n}| = 9$.

On the other hand,

- **Magnitude of $\vec{u}$:**

$$||\vec{u}|| = \sqrt{3^2 + (-2)^2 + 5^2} = \sqrt{9 + 4 + 25} = \sqrt{38}$$

- **Magnitude of $\vec{n}$:**

$$||\vec{n}|| = \sqrt{2^2 + 1^2 + 1^2} = \sqrt{4 + 1 + 1} = \sqrt{6}$$

Therefore,

$$\sin(\theta) = \frac{9}{\sqrt{38}\cdot\sqrt{6}} = \frac{9}{\sqrt{228}} = \frac{9}{2\sqrt{57}},$$

and the exact angle is given by

$$\theta = \arcsin\left(\frac{9}{2\sqrt{57}}\right) = arcsin\left(\frac{9\sqrt{57}}{2\cdot 3\cdot 19}\right) = arcsin\left(\frac{3\sqrt{57}}{38}\right)$$