ROMANIAN MATHEMATICAL MAGAZINE

JP.611 Find the angle between the real plans:

$$P_1: 2x + y + 3z - 1 = 0$$

$$P_2: 3x - 2y + z + 1 = 0$$

Proposed by Daniel Sitaru – Romania

Solution 1 by proposer

The normal vectors of P_1, P_2 are:

$$\overrightarrow{u_1}(2,1,3) = 2\vec{\imath} + \vec{\jmath} + 3\vec{k}$$

$$\overrightarrow{u_2}(3,-2,1) = 3\vec{\imath} - 2\vec{\jmath} + \vec{k}$$

$$\overrightarrow{u_1} \cdot \overrightarrow{u_2} = 2 \cdot 3 + 1 \cdot (-2) + 3 \cdot 1 = 6 - 2 + 3 = 7$$

$$|\overrightarrow{u_1}| = \sqrt{2^2 + 1^2 + 3^2} = \sqrt{14}$$

$$|\overrightarrow{u_2}| = \sqrt{3^2 + (-2)^2 + 1} = \sqrt{14}$$

$$\cos\left(\sphericalangle(\overrightarrow{u_1}, \overrightarrow{u_2})\right) = \frac{\overrightarrow{u_1} \cdot \overrightarrow{u_2}}{|\overrightarrow{u_1}| \cdot |\overrightarrow{u_2}|} = \frac{7}{\sqrt{14} \cdot \sqrt{14}} = \frac{7}{14} = \frac{1}{2}$$

$$\sphericalangle(\overrightarrow{u_1}, \overrightarrow{u_2}) = \sphericalangle(P_1, P_2) = \frac{\pi}{3}$$

Solution 2 by José Luis Díaz – Barrero – Spain

The acute angle θ between two planes is equal to the angle between their normal vectors, which is given by the formula:

$$\cos\theta = \frac{|\overrightarrow{n_1} \cdot \overrightarrow{n_2}|}{\left||\overrightarrow{n_1}|\right|\left||\overrightarrow{n_2}|\right|}$$

The normal vector coefficients correspond directly to the coefficients of x, y and z in each plane equation:

$$\overrightarrow{n_1} = (2,1,3)$$

$$\overrightarrow{n_2} = (3,-2,1)$$

$$\overrightarrow{n_1} \cdot \overrightarrow{n_2} = (2)(3) + (1)(-2) + (3)(1) = 6 - 2 + 3 = 7$$

$$\left||\overrightarrow{n_1}|\right| = \sqrt{2^2 + 1^2 + 3^2} = \sqrt{4 + 1 + 9} = \sqrt{14}$$

$$\left||\overrightarrow{n_2}|\right| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{9 + 4 + 1} = \sqrt{14}$$

Substituting the values back into the cosine formula:

$$\cos\theta = \frac{|7|}{\sqrt{14}\cdot\sqrt{14}} = \frac{7}{14} = \frac{1}{2}$$

Taking the inverse cosine ($\arccos$):

$$\theta = \arccos\left(\frac{1}{2}\right) = 60^\circ \quad \text{(or } \frac{\pi}{3} \text{ radians)}$$

Thus, the angle between the two planes is 60°.