

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$\frac{r_a^3}{r_b^2 - r_b r_c + r_c^2} + \frac{r_b^3}{r_c^2 - r_a r_c + r_a^2} + \frac{r_c^3}{r_a^2 - r_b r_a + r_b^2} \geq 9r$$

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$$\begin{aligned} & \frac{r_a^3}{r_b^2 - r_b r_c + r_c^2} + \frac{r_b^3}{r_c^2 - r_a r_c + r_a^2} + \frac{r_c^3}{r_a^2 - r_b r_a + r_b^2} = \\ & = \frac{r_a^3(r_b + r_c)}{r_b^3 + r_c^3} + \frac{r_b^3(r_a + r_c)}{r_a^3 + r_c^3} + \frac{r_c^3(r_b + r_a)}{r_b^3 + r_a^3} \end{aligned}$$

The last expression satisfies the conditions of the Walter Janous inequality

$$x, y, z, a, b, c > 0$$

$$\frac{x}{y+z}(b+c) + \frac{y}{x+z}(a+c) + \frac{z}{y+x}(b+a) \geq \sqrt{3(ab+bc+ac)}$$

Here $x = r_a^3, y = r_b^3, z = r_c^3, a = r_a, b = r_b, c = r_c$. Then :

$$\begin{aligned} & \frac{r_a^3(r_b + r_c)}{r_b^3 + r_c^3} + \frac{r_b^3(r_a + r_c)}{r_a^3 + r_c^3} + \frac{r_c^3(r_b + r_a)}{r_b^3 + r_a^3} \geq \\ & \geq \sqrt{3(r_a r_b + r_c r_b + r_a r_c)} = \sqrt{3 \cdot s^2} = \sqrt{3}s \geq \sqrt{3} \cdot 3\sqrt{3}r = 9r \end{aligned}$$

Equality holds for : $a = b = c$.