

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$\frac{r_a^3}{r_a^2 + r_a r_b + r_b^2} + \frac{r_b^3}{r_b^2 + r_b r_c + r_c^2} + \frac{r_c^3}{r_c^2 + r_c r_a + r_a^2} \geq \frac{3R}{2}$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Adgozel Adgozalov-Azerbaijan

$$\begin{aligned} \sum_{cyc} \frac{r_a^3}{r_a^2 + r_a r_b + r_b^2} &= \sum_{cyc} \frac{r_a^4}{r_a^3 + r_a^2 r_b + r_a r_b^2} \\ &\stackrel{\text{BERGSTROM}}{\geq} \frac{(\sum r_a^2)^2}{\sum_{cyc} (r_a^3 + r_a^2 r_b + r_a r_b^2)} = \\ &= \frac{(\sum r_a^2)^2}{(\sum r_a)(\sum r_a^2)} = \\ &= \frac{\sum r_a^2}{\sum r_a} \geq \frac{\frac{27}{4} R^2}{\frac{9}{2} R} = \frac{3R}{2} \end{aligned}$$

Equality holds for $a = b = c$.