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In $\triangle ABC$ the following relationship holds:

$$\frac{\sin^3 \frac{A}{2}}{\sin^2 \frac{A}{2} + \sin \frac{A}{2} \sin \frac{B}{2} + \sin^2 \frac{B}{2}} + \frac{\sin^3 \frac{B}{2}}{\sin^2 \frac{B}{2} + \sin \frac{B}{2} \sin \frac{C}{2} + \sin^2 \frac{C}{2}} + \frac{\sin^3 \frac{C}{2}}{\sin^2 \frac{C}{2} + \sin \frac{C}{2} \sin \frac{A}{2} + \sin^2 \frac{A}{2}} \geq \frac{1}{2}$$

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Solution by Adgozel Adgozalov-Azerbaijan

$$\begin{aligned} \sum_{cyc} \frac{\sin^3 \frac{A}{2}}{\sin^2 \frac{A}{2} + \sin \frac{A}{2} \sin \frac{B}{2} + \sin^2 \frac{B}{2}} &= \sum_{cyc} \frac{\sin^4 \frac{A}{2}}{\sin^3 \frac{A}{2} + \sin^2 \frac{A}{2} \sin \frac{B}{2} + \sin \frac{A}{2} \sin^2 \frac{B}{2}} \geq \\ &\stackrel{\text{BERGSTROM}}{\geq} \frac{\left(\sum \sin^2 \frac{A}{2}\right)^2}{\sum_{cyc} \left(\sin^3 \frac{A}{2} + \sin^2 \frac{A}{2} \sin \frac{B}{2} + \sin \frac{A}{2} \sin^2 \frac{B}{2}\right)} = \\ &= \frac{\left(\sum \sin^2 \frac{A}{2}\right)^2}{\left(\sum \sin \frac{A}{2}\right) \left(\sum \sin^2 \frac{A}{2}\right)} = \\ &= \frac{\sum \sin^2 \frac{A}{2}}{\sum \sin \frac{A}{2}} \geq \frac{\frac{3}{4}}{\frac{3}{2}} = \frac{1}{2} \end{aligned}$$

Equality holds for $a = b = c$.