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In any ΔABC the following relationship holds :

$$\frac{r_a}{a} + \frac{r_b}{b} + \frac{r_c}{c} \leq \frac{3(r_a + r_b + r_c)}{a + b + c}$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Soumava Chakraborty-Kolkata-India

$$\sum_{\text{cyc}} \frac{r_a}{a} = \sum_{\text{cyc}} \frac{s \tan \frac{A}{2}}{4R \tan \frac{A}{2} \cos^2 \frac{A}{2}} = \frac{s}{4R} \cdot \sum_{\text{cyc}} \sec^2 \frac{A}{2} = \frac{s}{4R} \cdot \frac{s^2 + (4R + r)^2}{s^2}$$

$$\stackrel{\text{Doucet}}{\leq} \frac{1}{4R} \cdot \frac{\frac{(4R+r)^2}{3} + (4R + r)^2}{s} \stackrel{\text{Euler}}{\leq} \frac{(4R + r)(9R)}{3R(a + b + c)} = \frac{3(r_a + r_b + r_c)}{a + b + c},$$

" = " iff ΔABC is equilateral (QED)