ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{m_a+m_b}{\sqrt{m_a^2+m_am_b+m_b^2}}+\frac{m_c+m_b}{\sqrt{m_c^2+m_cm_b+m_b^2}}+\frac{m_a+m_c}{\sqrt{m_a^2+m_am_c+m_c^2}}\ge\frac{4\sqrt{3}r}{R}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\sum_{cyc}\frac{m_a+m_b}{\sqrt{m_a^2+m_am_b+m_b^2}}=\sum_{cyc}\frac{(m_a+m_b)^{\frac{3}{2}}}{\sqrt{(m_a^2+m_am_b+m_b^2)(m_a+m_b)}}\overset{Radon}{\geq}$$

$$\frac{\left(2(m_a+m_b+m_c)\right)^{\frac{3}{2}}}{\left(\sum_{cyc}(m_a^2+m_am_b+m_b^2)(m_a+m_b)\right)^{\frac{1}{2}}}\ge\frac{\left(2(m_a+m_b+m_c)\right)^{\frac{3}{2}}}{\left(\frac{27}{4}R^2.2(m_a+m_b+m_c)\right)^{\frac{1}{2}}}=$$

$$=\frac{2\sqrt{2}}{\frac{3\sqrt{3}R}{\sqrt{2}}}(m_a+m_b+m_c)\overset{Gotman}{\geq}\frac{4.9r}{3\sqrt{3}R}=\frac{4\sqrt{3}r}{R}$$

$$\therefore m_a^2+m_am_b+m_b^2=\frac{3}{4}(a^2+b^2+c^2)\overset{Leibniz}{\leq}\frac{3}{4}.9R^2=\frac{27}{4}R^2$$

$$Equality\ \ holds\ \ for:\quad a=b=c$$