ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{r_a+r_b}{\sqrt{r_a^2+r_ar_b+r_b^2}}+\frac{r_c+r_b}{\sqrt{r_c^2+r_cr_b+r_b^2}}+\frac{r_a+r_c}{\sqrt{r_a^2+r_ar_c+r_c^2}}\ge\frac{4\sqrt{3}r}{\sqrt{3R^2-8r^2}}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\sum_{cyc}\frac{r_a+r_b}{\sqrt{r_a^2+r_ar_b+r_b^2}}=\sum_{cyc}\frac{(r_a+r_b)^{\frac{3}{2}}}{\sqrt{(r_a^2+r_ar_b+r_b^2)(r_a+r_b)}}\overset{Radon}{\ge}$$

$$\ge\frac{\left(2(r_a+r_b+r_c)\right)^{\frac{3}{2}}}{\left(\sum_{cyc}(r_a^2+r_ar_b+r_b^2)(r_a+r_b)\right)^{\frac{1}{2}}}\ge\frac{\left(2(r_a+r_b+r_c)\right)^{\frac{3}{2}}}{\left(\frac{27}{4}R^2.2(r_a+r_b+r_c)\right)^{\frac{1}{2}}}=$$

$$=\frac{2\sqrt{2}}{\frac{3\sqrt{3}R}{\sqrt{2}}}(r_a+r_b+r_c)=\frac{4(4R+r)}{3\sqrt{3}R}\overset{Euler}{\ge}\frac{4.9r}{3\sqrt{3}R}=\frac{4\sqrt{3}r}{R}\ge\frac{4\sqrt{3}r}{\sqrt{3R^2-8r^2}}$$

$$\therefore R\le\sqrt{3R^2-8r^2}\to R^2\le 3R^2-8r^2\to R^2\ge 4r^2\to R\ge 2r\ (True)$$

$$Equality\ \ holds\ \ for:\ \ a=b=c$$