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In any ΔABC the following relationship holds :

$$\sum_{cyc} \frac{\sin^2 B + \sin^2 C}{(\sin A \sin B)(\sin A \sin C + \sin^2 B)} \geq 4$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\begin{aligned} \text{LHS} &\stackrel{\text{CBS}}{\geq} \sum_{cyc} \frac{\sin^2 B + \sin^2 C}{(\sin A \sin B) \left(\sqrt{(\sin^2 A + \sin^2 B)(\sin^2 B + \sin^2 C)} \right)} \stackrel{\text{AM-GM}}{\geq} \\ &\frac{3}{(\sqrt[3]{\sin A \sin B \sin C})^2} \stackrel{\text{AM-GM}}{\geq} \frac{3}{\left(\frac{\sum_{cyc} \sin A}{3} \right)^2} = \frac{27R^2}{s^2} \stackrel{\text{Mitrinovic}}{\geq} 4 \quad \forall \Delta ABC, \\ &\quad \text{"=" iff } \Delta ABC \text{ is equilateral (QED)} \end{aligned}$$