ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{A}{a}+\frac{B}{b}+\frac{C}{c}\geq\frac{\pi\sqrt{3}}{3R}$$

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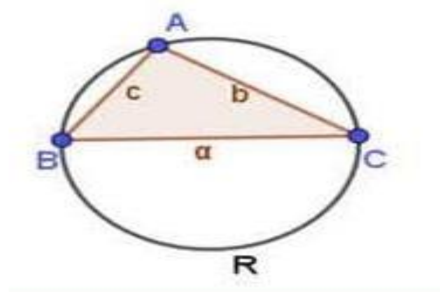

It is known that in ΔABC holds: $\frac{\sin A}{A}\cdot\frac{\sin B}{B}\cdot\frac{\sin C}{C}\leq\left(\frac{3\sqrt{3}}{2\pi}\right)^3$

$$\frac{A}{a}+\frac{B}{b}+\frac{C}{c}\overset{AM-GM}{\geq}3\sqrt[3]{\frac{A\cdot B\cdot C}{a\cdot b\cdot c}}=3\sqrt[3]{\frac{1}{(2R)^3}\cdot\left(\frac{2\pi}{3\sqrt{3}}\right)^3}=3\cdot\frac{1}{2R}\cdot\frac{2\pi}{3\sqrt{3}}=\frac{\pi\sqrt{3}}{3R}$$

$$Equality\ holds\ if\ a=b=c$$