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In $\triangle ABC$ the following relationship holds:

$$h_a h_b h_c \leq \sqrt[4]{27} \cdot F \sqrt{F}$$

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Solution by Daniel Sitaru – Romania

$$\begin{aligned} h_a h_b h_c &= \frac{2F}{a} \cdot \frac{2F}{b} \cdot \frac{2F}{c} = \frac{8F^3}{abc} = \frac{8F^3}{4RF} = \frac{2F^2}{R} \leq \frac{2F^2}{2r} = \frac{F^2}{r} = \frac{F \cdot F}{2} = \frac{F \cdot rs}{r} = \\ &= Fs = F\sqrt{s^2} = F\sqrt{s \cdot s} \stackrel{\text{MITRINOVIC}}{\geq} F\sqrt{s \cdot 3\sqrt{3}r} = F\sqrt{F \cdot \sqrt{27}} = \sqrt[4]{27} F \sqrt{F} \end{aligned}$$

Equality holds for $a = b = c$.