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In $\triangle ABC$ the following relationship holds:

$$r_a r_b r_c \geq \sqrt[4]{27} \cdot F \sqrt{F}$$

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Solution by Daniel Sitaru – Romania

$$\begin{aligned} r_a r_b r_c &= \frac{F}{s-a} \cdot \frac{F}{s-b} \cdot \frac{F}{s-c} = \frac{F^3 s}{s(s-a)(s-b)(s-c)} = \\ &= \frac{F^3 s}{F^2} = F s = F \sqrt{s^2} = F \sqrt{s \cdot s} \geq \\ &\stackrel{\text{MITRINOVIC}}{\geq} F \sqrt{s \cdot 3\sqrt{3}r} = F \sqrt{F \cdot \sqrt{27}} = \sqrt[4]{27} F \sqrt{F} \end{aligned}$$

Equality holds for $a = b = c$.