

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$\frac{r_a}{r_b + r_c} + \frac{r_b}{r_a + r_c} + \frac{r_c}{r_b + r_a} \leq \frac{3R}{4r}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\begin{aligned} \frac{r_a}{r_b + r_c} + \frac{r_b}{r_a + r_c} + \frac{r_c}{r_b + r_a} &\leq \frac{r_a}{4} \left(\frac{1}{r_b} + \frac{1}{r_c} \right) + \frac{r_b}{4} \left(\frac{1}{r_a} + \frac{1}{r_c} \right) + \frac{r_c}{4} \left(\frac{1}{r_b} + \frac{1}{r_a} \right) = \\ &= \frac{1}{4} \left(\frac{r_a}{r_b} + \frac{r_a}{r_c} + \frac{r_b}{r_c} + \frac{r_b}{r_a} + \frac{r_c}{r_b} + \frac{r_c}{r_a} \right) = \frac{1}{4} \left(\left(\frac{r_a}{r_b} + \frac{r_b}{r_c} + \frac{r_c}{r_a} \right) + \left(\frac{r_b}{r_a} + \frac{r_a}{r_c} + \frac{r_c}{r_b} \right) \right) \stackrel{\text{Popovici}}{\geq} \\ &\geq \frac{1}{4} \left(\frac{3R}{2r} + \frac{3R}{2r} \right) = \frac{3R}{4r} \end{aligned}$$

Equality holds if $a = b = c$