

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$\cos A \cos B \cos C \leq \frac{r}{4R}$$

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$$\cos A \cos B \cos C = \frac{s^2 - (2R + r)^2}{4R^2} \leq \frac{r}{4R} \Leftrightarrow$$

$$\Leftrightarrow s^2 - (4R^2 + 4Rr + r^2) \leq Rr$$

$$s^2 \leq 4R^2 + 4Rr + r^2 + Rr = 4R^2 + 5Rr + r^2$$

$$s^2 \stackrel{\text{GERRETSEN}}{\leq} 4R^2 + 4Rr + 3r^2 \leq 4R^2 + 5Rr + r^2 \Leftrightarrow 2r^2 \leq Rr \Leftrightarrow R \geq 2r \quad (\text{Euler})$$

Equality holds for $A = B = C$.