ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sin A \sin B \sin C \geq \frac{3\sqrt{3}r^3}{R^3}$$

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$$\sin A \sin B \sin C = \frac{a}{2R} \cdot \frac{b}{2R} \cdot \frac{c}{2R} = \frac{abc}{8R^3} \geq \frac{3\sqrt{3}r^3}{R^3}$$

$$\Leftrightarrow abc \geq 24\sqrt{3}r^3 \Leftrightarrow 4Rrs \geq 24\sqrt{3}r^3 \Leftrightarrow Rs \geq 6\sqrt{3}r^2 \quad \text{(to prove)}$$

$$Rs \overset{EULER}{\geq} 2r \cdot s \overset{MITRINOVIC}{\geq} 2r \cdot 3\sqrt{3}r = 6\sqrt{3}r^2$$