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In ΔABC the following relationship holds:

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{r_a}}+\frac{\sqrt{c}+\sqrt{b}}{\sqrt{r_b}}+\frac{\sqrt{a}+\sqrt{c}}{\sqrt{r_c}}\geq 2\sqrt[4]{108}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{r_a}}+\frac{\sqrt{c}+\sqrt{b}}{\sqrt{r_b}}+\frac{\sqrt{a}+\sqrt{c}}{\sqrt{r_c}}\overset{AM-GM}{\geq}$$

$$\geq\frac{2\sqrt[4]{ab}}{\sqrt{r_a}}+\frac{2\sqrt[4]{bc}}{\sqrt{r_b}}+\frac{2\sqrt[4]{ac}}{\sqrt{r_c}}\overset{AM-GM}{\geq}3\sqrt[3]{\frac{2\sqrt[4]{ab}.2\sqrt[4]{bc}.2\sqrt[4]{ac}}{\sqrt{r_a.r_b.r_c}}}=$$

$$=6\sqrt[6]{\frac{\text{abc}}{r_a.r_b.r_c}}=6\sqrt[6]{\frac{4R.F}{s^2.r}}=6\sqrt[6]{\frac{4R.sr}{s^2.r}}=6\sqrt[6]{\frac{4R}{s}}\overset{Mitrinovici}{\geq}$$

$$\geq 6\sqrt[6]{\frac{4}{s}.\frac{2s}{3\sqrt{3}}}=6\sqrt[6]{\left(\frac{2^3}{\sqrt{27}}\right)}=6\sqrt{\frac{2}{\sqrt{3}}}=2\sqrt[4]{108}$$

$$\textit{Equality holds for}:\quad a=b=c$$