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In ΔABC the following relationship holds:

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{h_c}}+\frac{\sqrt{b}+\sqrt{c}}{\sqrt{h_a}}+\frac{\sqrt{c}+\sqrt{a}}{\sqrt{h_b}}\ge 2\sqrt[4]{108}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$s^2 \overset{Mitrinovic}{\le} \frac{27}{4}R^2 \ or, \frac{R}{s}\ge\frac{2}{3\sqrt{3}}\ (1)$$

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{h_c}}+\frac{\sqrt{b}+\sqrt{c}}{\sqrt{h_a}}+\frac{\sqrt{c}+\sqrt{a}}{\sqrt{h_b}}=\sum\frac{\sqrt{a}+\sqrt{b}}{\sqrt{h_c}}=\sum\frac{\sqrt{a}+\sqrt{b}}{\sqrt{\frac{ab}{2R}}}=\sqrt{2R}\sum\left(\frac{1}{\sqrt{b}}+\frac{1}{\sqrt{a}}\right)=$$

$$=2\sqrt{2R}\sum\frac{1}{\sqrt{a}}=2\sqrt{2R}\sum\frac{1^{\frac{3}{2}}}{\sqrt{a}}\overset{Radon}{\ge}2\sqrt{2R}\frac{(1+1+1)^{\frac{3}{2}}}{\sqrt{a+b+c}}=2\sqrt{2R}\frac{(1+1+1)^{\frac{3}{2}}}{\sqrt{2s}}=$$

$$=2\sqrt{\frac{R}{s}3\sqrt{3}}\ge 2\sqrt{\frac{2}{3\sqrt{3}}\times 3\sqrt{3}}=2\sqrt{6\sqrt{3}}=2\sqrt[4]{36\times 3}=2\sqrt[4]{108}$$

Equality holds for an equilateral triangle.