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In $\triangle ABC$ the following relationship holds:

$$\frac{GA}{r_b} + \frac{GB}{r_c} + \frac{GC}{r_a} \geq 2$$

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Solution by Daniel Sitaru – Romania

$$\begin{aligned} \frac{GA}{r_b} + \frac{GB}{r_c} + \frac{GC}{r_a} &\geq 3 \sqrt[3]{\frac{GA \cdot GB \cdot GC}{r_a r_b r_c}} = \\ &= 3 \sqrt[3]{\frac{\frac{2}{3}m_a \cdot \frac{2}{3}m_b \cdot \frac{2}{3}m_c}{r_a r_b r_c}} = 3 \cdot \frac{2}{3} \sqrt[3]{\frac{m_a m_b m_c}{r_a r_b r_c}} \geq 2 \sqrt[3]{\frac{r_a r_b r_c}{r_a r_b r_c}} = 2 \end{aligned}$$

Equality holds for $a = b = c$.