ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{GA\cdot GB}{r_c^2}+\frac{GB\cdot GC}{r_a^2}+\frac{GC\cdot GA}{r_b^2}\ge\frac{4}{3}$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\frac{GA\cdot GB}{r_c^2}+\frac{GB\cdot GC}{r_a^2}+\frac{GC\cdot GA}{r_b^2}\ge$$

$$\ge 3\cdot\sqrt[3]{\left(\frac{GA\cdot GB\cdot GC}{r_ar_br_c}\right)^2}=3\sqrt[3]{\left(\frac{\frac{2}{3}m_a\cdot\frac{2}{3}m_b\cdot\frac{2}{3}m_c}{r_ar_br_c}\right)^2}=$$

$$=3\cdot\left(\frac{2}{3}\right)^2\cdot\sqrt[3]{\left(\frac{m_am_bm_c}{r_ar_br_c}\right)^2}=3\cdot\frac{4}{9}\cdot\sqrt[3]{\left(\frac{r_ar_br_c}{r_ar_br_c}\right)^2}=\frac{4}{3}$$

Equality holds for $a=b=c$.