ROMANIAN MATHEMATICAL MAGAZINE

**In $\Delta ABC$ the following relationship holds:**

$$\frac{a}{h_a \sin^2 A}+\frac{b}{h_b \sin^2 B}+\frac{c}{h_c \sin^2 C}\ge \frac{8\sqrt{3}}{3}$$

***Proposed by Nguyen Hung Cuong – Vietnam***

***Solution by Daniel Sitaru – Romania***

$$\frac{a}{h_a \sin^2 A}+\frac{b}{h_b \sin^2 B}+\frac{c}{h_c \sin^2 C}=\sum_{cyc}\frac{a}{h_a \sin^2 A}=\sum\frac{a}{\frac{2F}{a}\cdot\left(\frac{a}{2R}\right)^2}=\sum\frac{a\cdot 4R^2}{2F\cdot a}=$$

$$=\frac{4R^2}{2F}\cdot 3=\frac{12R^2}{2F}=\frac{6R^2}{rs}\overset{EULER}{\ge}\frac{6R\cdot 2r}{rs}=\frac{12R}{s}\overset{MITRINOVIC}{\ge}\frac{12R}{\frac{3\sqrt{3}R}{2}}=\frac{24}{3\sqrt{3}}=\frac{8}{\sqrt{3}}=\frac{8\sqrt{3}}{3}$$

**Equality holds for $a=b=c$.**