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In ΔABC the following relationship holds:

$$\frac{a}{w_b}+\frac{b}{w_c}+\frac{c}{w_a}\geq 2\sqrt{3}$$

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Solution by Daniel Sitaru – Romania

$$\frac{a}{w_b}+\frac{b}{w_c}+\frac{c}{w_a}\overset{AM-GM}{\geq} 3\cdot\sqrt[3]{\frac{abc}{w_aw_bw_c}}\geq 3\sqrt[3]{\frac{4RF}{\sqrt{s(s-a)}\cdot\sqrt{s(s-b)}\cdot\sqrt{s(s-c)}}}=$$

$$=3\sqrt[3]{\frac{4RF}{s\cdot F}}=3\sqrt[3]{\frac{4R}{s}}\overset{MITRINOVIC}{\geq} 3\sqrt[3]{\frac{4\cdot\frac{2}{3\sqrt{3}}s}{s}}=3\sqrt[3]{\frac{8}{3\sqrt{3}}}=3\sqrt[3]{\left(\frac{2}{\sqrt{3}}\right)^2}=$$

$$=3\cdot\frac{2}{\sqrt{3}}=\frac{6}{\sqrt{3}}=\frac{6\sqrt{3}}{3}=2\sqrt{3}$$

Equality holds for $a=b=c$.