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In ΔABC the following relationship holds:

$$\frac{r_a^2}{h_b h_c}+\frac{r_b^2}{h_c h_a}+\frac{r_c^2}{h_a h_b}\geq 3$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\frac{r_a^2}{h_b h_c}+\frac{r_b^2}{h_c h_a}+\frac{r_c^2}{h_a h_b}\geq 3\sqrt[3]{\left(\frac{r_a r_b r_c}{h_a h_b h_c}\right)^2}=3\sqrt[3]{\left(\frac{\frac{F}{s-a}\cdot\frac{F}{s-b}\cdot\frac{F}{s-c}}{\frac{2F}{a}\cdot\frac{2F}{b}\cdot\frac{2F}{c}}\right)^2}=$$

$$=\frac{3}{4}\sqrt[3]{\left(\frac{abc}{(s-a)(s-b)(s-c)}\right)^2}=\frac{3}{4}\sqrt[3]{\left(\frac{abcs}{s(s-a)(s-b)(s-c)}\right)^2}=\frac{3}{4}\sqrt[3]{\left(\frac{4RFs}{F^2}\right)^2}=$$

$$=\frac{3}{4}\sqrt[3]{\left(\frac{4Rs}{rs}\right)^2}=\frac{3}{4}\sqrt[3]{\left(\frac{4R}{r}\right)^2}\overset{EULER}{\geq}\frac{3}{4}\sqrt[3]{\left(\frac{8r}{r}\right)^2}=\frac{3}{4}\sqrt[3]{64}=\frac{3}{4}\cdot 4=3$$

Equality holds for $a=b=c$.