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In ΔABC the following relationship holds:

$$\frac{a^2}{w_b w_c}+\frac{b^2}{w_c w_a}+\frac{c^2}{w_a w_b}\geq 4$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\frac{a^2}{w_b w_c}+\frac{b^2}{w_c w_a}+\frac{c^2}{w_a w_a}\geq 3\sqrt[3]{\left(\frac{abc}{w_a w_b w_c}\right)^2}\geq 3\sqrt[3]{\left(\frac{abc}{\sqrt{s(s-a)}\cdot\sqrt{s(s-b)}\cdot\sqrt{s(s-c)}}\right)^2}=$$

$$=3\sqrt[3]{\left(\frac{abc}{sF}\right)^2}=3\sqrt[3]{\left(\frac{4RF}{sF}\right)^2}=3\sqrt[3]{\left(\frac{4R}{s}\right)^2}\overset{MITRINOVIC}{\geq}3\sqrt[3]{\left(\frac{4R}{\frac{3\sqrt{3}}{2}R}\right)^2}=3\sqrt[3]{\left(\frac{8}{3\sqrt{3}}\right)^2}=$$

$$=3\sqrt[3]{\frac{64}{27}}=3\cdot\frac{4}{3}=4$$

Equality holds for $a=b=c$.