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In ΔABC the following relationship holds:

$$\frac{a}{r_b^2}+\frac{b}{r_c^2}+\frac{c}{r_a^2}\geq\frac{4\sqrt{3}}{3R}$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\frac{a}{r_b^2}+\frac{b}{r_c^2}+\frac{c}{r_a^2}\geq 3\sqrt[3]{\frac{abc}{(r_ar_br_c)^2}}=3\sqrt[3]{\frac{abc}{\left(\frac{F}{s-a}\cdot\frac{F}{s-b}\cdot\frac{F}{s-c}\right)^2}}=3\sqrt[3]{\frac{abc}{\left(\frac{F^3s}{F^2}\right)^2}}=$$

$$=3\sqrt[3]{\frac{4RF}{(Fs)^2}}=3\sqrt[3]{\frac{4RF}{F^2s^2}}=3\sqrt[3]{\frac{4R}{Fs^2}}=3\sqrt[3]{\frac{4R}{rs^3}}=\frac{3}{s}\sqrt[3]{\frac{4R}{r}}\overset{EULER}{\geq}\frac{3}{s}\sqrt[3]{\frac{8r}{r}}=$$

$$=\frac{3}{s}\cdot\sqrt[3]{8}\overset{MITRINOVIC}{\geq}\frac{3}{\frac{3\sqrt{3}R}{2}}\cdot 2=\frac{4}{\sqrt{3}R}=\frac{4\sqrt{3}}{3R}$$

Equality holds for $a=b=c$.