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In any ΔABC the following relationship holds :

$$\frac{a \cot \frac{A}{2}}{r_a} + \frac{b \cot \frac{B}{2}}{r_b} + \frac{c \cot \frac{C}{2}}{r_c} \geq 6$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Soumava Chakraborty-Kolkata-India

$$\begin{aligned} \text{LHS} &\stackrel{\text{AM-GM}}{\geq} 3 \cdot \sqrt[3]{\frac{4Rrs \cdot s^3}{(r_a r_b r_c)^2}} = 3 \cdot \sqrt[3]{\frac{4Rrs^4}{r^2 s^4}} \stackrel{\text{Euler}}{\geq} 3 \cdot \sqrt[3]{\frac{8r}{r}} = 6 \quad \forall \text{ } \Delta ABC, \\ &'' = '' \text{ iff } \Delta ABC \text{ is equilateral (QED)} \end{aligned}$$