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In any acute ΔABC the following relationship holds :

$$\frac{a \tan A}{r_a} + \frac{b \tan B}{r_b} + \frac{c \tan C}{r_c} \geq 6$$

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For an acute ΔABC , $\sum_{\text{cyc}} \tan A \stackrel{\text{Jensen}}{\geq} 3\sqrt{3} \Rightarrow \prod_{\text{cyc}} \tan A \geq 3\sqrt{3}$

$$\begin{aligned} \therefore \text{LHS} &\stackrel{\text{AM-GM}}{\geq} 3 \cdot \sqrt[3]{\frac{4Rrs \cdot 3\sqrt{3}}{rs^2}} \stackrel{\text{Mitrinovic}}{\geq} 3 \cdot \sqrt[3]{4 \cdot \frac{2}{3\sqrt{3}} \cdot 3\sqrt{3}} = 6 \quad \forall \text{ acute } ABC, \\ &\text{" = " iff } \Delta ABC \text{ is equilateral (QED)} \end{aligned}$$