

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$\frac{a}{r_a \cos^2 \frac{A}{2}} + \frac{b}{r_b \cos^2 \frac{B}{2}} + \frac{c}{r_c \cos^2 \frac{C}{2}} \geq \frac{8\sqrt{3}}{3}$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\begin{aligned} \frac{a}{r_a \cos^2 \frac{A}{2}} + \frac{b}{r_b \cos^2 \frac{B}{2}} + \frac{c}{r_c \cos^2 \frac{C}{2}} &= \sum_{cyc} \frac{a}{r_a \cos^2 \frac{A}{2}} = \\ &= \sum_{cyc} \frac{a}{\frac{F}{s-a} \cdot \frac{s(s-a)}{bc}} = \sum_{cyc} \frac{abc}{Fs} = \frac{3abc}{Fs} = \frac{3 \cdot 4RF}{Fs} = \frac{12R}{s} \geq \frac{12 \cdot \frac{2}{3\sqrt{3}}s}{s} = \frac{8}{\sqrt{3}} = \frac{8\sqrt{3}}{3} \end{aligned}$$

Equality holds for $a = b = c$.