ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{a}{h_b^3}+\frac{b}{h_c^3}+\frac{c}{h_a^3}\geq\frac{8\sqrt{3}}{9R^2}$$

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Solution by Daniel Sitaru – Romania

$$\frac{a}{h_b^3}+\frac{b}{h_c^3}+\frac{c}{h_c^3}\overset{AM-GM}{\geq}3\sqrt[3]{\frac{abc}{(h_ah_bh_c)^3}}=\frac{3}{h_ah_bh_c}\cdot\sqrt[3]{4Rrs}\overset{EULER}{\geq}\frac{3}{\frac{2F}{a}\cdot\frac{2F}{b}\cdot\frac{2F}{c}}\sqrt[3]{8r^2s}\geq$$

$$\overset{MITRINOVIC}{\geq}\frac{3abc}{8F^3}\cdot 2\sqrt[3]{r^2\cdot 3\sqrt{3}r}=\frac{3abc}{4F^3}\cdot\sqrt{3}\cdot r=\frac{3\sqrt{3}\cdot 4RFr}{4F^3}=\frac{3\sqrt{3}Rr}{F^2}=$$

$$=\frac{3\sqrt{3}Rr}{r^2s^2}=\frac{3\sqrt{3}R}{rs^2}\geq\frac{8\sqrt{3}}{9R^2}\Leftrightarrow 27\sqrt{3}R^3\geq 8\sqrt{3}rs^2$$

$$27R^3\geq 8rs^2$$

$27R^3=27R^2\cdot 2r\geq 8rs^2\Leftrightarrow 27R^2\geq 4s^2\Leftrightarrow s^2\leq\frac{27R^2}{4}\Leftrightarrow s\leq\frac{3\sqrt{3}}{2}R$ **(MITRINOVIC)**

Equality holds for $a=b=c$.