In ΔABC the following relationship holds:

$$\frac{a}{h_b^2}+\frac{b}{h_c^2}+\frac{c}{h_c^2}\ge\frac{4\sqrt{3}}{3R}$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\frac{a}{h_b^2}+\frac{b}{h_c^2}+\frac{c}{h_a^2}\overset{AM-GM}{\ge}3\sqrt[3]{\frac{abc}{(h_ah_bh_c)^2}}=3\sqrt[3]{\frac{abc}{\left(\frac{2F}{a}\cdot\frac{2F}{b}\cdot\frac{2F}{c}\right)^2}}=3\sqrt[3]{\frac{(abc)^3}{64F^6}}=$$

$$=3\cdot\frac{abc}{4F^2}=3\cdot\frac{4RF}{4F^2}=\frac{3R}{F}=\frac{3R}{rs}\overset{MITRINOVIC}{\ge}\frac{3R}{r\cdot\frac{3\sqrt{3}}{2}R}=\frac{2}{r\sqrt{3}}\ge\frac{4\sqrt{3}}{3R}\Leftrightarrow$$

$$\Leftrightarrow 6R\ge 12r\Leftrightarrow R\ge 2r\quad\text{(Euler)}$$

Equality holds for $a=b=c$.