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In acute ΔABC the following relationship holds:

$$\frac{a}{r_a \cos A}+\frac{b}{r_b \cos B}+\frac{c}{r_c \cos C}\geq 4\sqrt{3}$$

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We will use the next known result:

$$\cos A\cos B\cos C\leq \frac{1}{8}\quad (1)$$

$$\frac{a}{r_a \cos A}+\frac{b}{r_b \cos B}+\frac{c}{r_c \cos C}\geq 3\sqrt[3]{\frac{abc}{r_ar_br_c \cos A\cos B\cos C}}\geq$$

$$\overset{(1)}{\geq} 3\sqrt[3]{\frac{abc}{\frac{F}{s-a}\cdot\frac{F}{s-b}\cdot\frac{F}{s-c}\cdot\frac{1}{8}}}=3\sqrt[3]{\frac{8abc(s-a)(s-b)(s-c)}{F^3}}=$$

$$=3\sqrt[3]{\frac{8abc\cdot F^2}{sF^3}}=6\sqrt[3]{\frac{4RF}{sF}}=6\sqrt[3]{\frac{4R}{s}}\overset{MITRINOVIC}{\geq}6\sqrt[3]{\frac{4\cdot\frac{2}{3\sqrt{3}}s}{s}}=6\sqrt[3]{\left(\frac{2}{\sqrt{3}}\right)^3}=$$

$$=6\cdot\frac{2}{\sqrt{3}}=\frac{12\sqrt{3}}{3}=4\sqrt{3}$$

Equality holds for $a=b=c$.