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In $\triangle ABC$ the following relationship holds:

$$\frac{ab}{c} + \frac{bc}{a} + \frac{ca}{b} \geq 6\sqrt{3}r$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\begin{aligned} \frac{ab}{c} + \frac{bc}{a} + \frac{ca}{b} &\stackrel{AM-GM}{\geq} 3 \sqrt[3]{\frac{ab}{c} \cdot \frac{bc}{a} \cdot \frac{ca}{b}} = 3\sqrt[3]{abc} = 3\sqrt[3]{4Rrs} \stackrel{EULER}{\geq} \\ &\geq 3\sqrt[3]{4 \cdot 2r \cdot rs} \stackrel{MITRINOVIC}{\geq} 3 \cdot 2 \sqrt[3]{r^2 \cdot 3\sqrt{3}r} = 6\sqrt[3]{(r\sqrt{3})^3} = 6\sqrt{3}r \end{aligned}$$

Equality holds for $a = b = c$.