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In ΔABC the following relationship holds:

$$\frac{a\sin A}{h_a}+\frac{b\sin B}{h_b}+\frac{c\sin C}{h_c}\geq 3$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\frac{a\sin A}{h_a}+\frac{b\sin B}{h_b}+\frac{c\sin C}{h_c}=\sum_{cyc}\frac{a\sin A}{h_a}=$$

$$=\sum_{cyc}\frac{a\cdot\frac{a}{2R}}{\frac{2F}{a}}=\frac{1}{4RF}\sum_{cyc}a^3\overset{AM-GM}{\geq}\frac{1}{abc}\cdot 3\sqrt[3]{(abc)^3}=\frac{3abc}{abc}=3$$

Equality holds for $a=b=c$.