ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum_{cyc} \frac{\sin A}{h_b} \ge \frac{\sqrt{3}}{R}$$

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Solution by Daniel Sitaru – Romania

$$\sum_{cyc} \frac{\sin A}{h_b} = \sum_{cyc} \frac{\frac{a}{2R}}{\frac{2F}{b}} = \frac{1}{4RF} \sum_{cyc} ab =$$

$$= \frac{1}{4Rrs} \cdot (s^2 + r^2 + 4Rr) \overset{MITRINOVIC}{\ge} \frac{1}{4Rr \cdot \frac{3\sqrt{3}}{2} R} (s^2 + r^2 + 4Rr) \ge$$

$$\overset{GERRETSEN}{\ge} \frac{1}{6\sqrt{3}R^2 r} (16Rr - 5r^2 + r^2 + 4Rr) = \frac{1}{6\sqrt{3}R^2 r} (20Rr - 4r^2) =$$

$$= \frac{2r(10R - 2r)}{6\sqrt{3}R^2 r} = \frac{10R - 2r}{3\sqrt{3}R^2} \ge \frac{\sqrt{3}}{R} \Leftrightarrow 10R - 2r \ge 9R \Leftrightarrow R \ge 2r \quad \text{(Euler)}$$

Equality holds for $a = b = c$.