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In ΔABC the following relationship holds:

$$\frac{a}{h_a \sin A}+\frac{b}{h_b \sin B}+\frac{c}{h_c \sin C}\geq 4$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\frac{a}{h_a \sin A}+\frac{b}{h_b \sin B}+\frac{c}{h_c \sin C}=\sum_{cyc}\frac{a}{h_a \sin A}=$$

$$=\sum_{cyc}\frac{a}{\frac{2F}{a}\cdot\frac{a}{2R}}=\frac{R}{F}\sum_{cyc}a=\frac{R}{F}\cdot 2s=\frac{R}{rs}\cdot 2s=\frac{2R}{r}\overset{EULER}{\geq}\frac{2\cdot 2r}{r}=4$$

Equality holds for $a=b=c$.