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In $\triangle ABC$ the following relationship holds:

$$\frac{ab}{\sin \frac{C}{2}} + \frac{bc}{\sin \frac{A}{2}} + \frac{ca}{\sin \frac{B}{2}} \geq 72r^2$$

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Solution by Daniel Sitaru – Romania

$$\begin{aligned} \frac{ab}{\sin \frac{C}{2}} + \frac{bc}{\sin \frac{A}{2}} + \frac{ca}{\sin \frac{B}{2}} &= \sum_{cyc} \frac{ab}{\sin \frac{C}{2}} \stackrel{CEBYSHEV}{\geq} \frac{1}{3} \sum_{cyc} ab \cdot \sum_{cyc} \frac{1}{\sin \frac{C}{2}} \stackrel{JENSEN}{\geq} \\ &\geq \frac{1}{3} (s^2 + r^2 + 4Rr) \cdot \frac{3}{\sin \left(\frac{A+B+C}{6} \right)} \stackrel{MITRINOVIC}{\geq} (27r^2 + r^2 + 4Rr) \cdot \frac{1}{\sin \frac{\pi}{6}} \stackrel{EULER}{\geq} \\ &\geq (28r^2 + 4 \cdot 2r \cdot r) \cdot \frac{1}{\frac{1}{2}} = 2(28r^2 + 8r^2) = 2 \cdot 36r^2 = 72r^2 \end{aligned}$$

Equality holds for $a = b = c$.