ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{\sin A}{r_b}+\frac{\sin B}{r_c}+\frac{\sin C}{r_a}\geq\frac{\sqrt{3}}{R}$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\frac{\sin A}{r_b}+\frac{\sin B}{r_c}+\frac{\sin C}{r_a}\geq 3\sqrt[3]{\frac{\sin A\sin B\sin C}{r_a r_b r_c}}=$$

$$=3\sqrt[3]{\frac{a}{2R}\cdot\frac{b}{2R}\cdot\frac{c}{2R}\cdot\frac{(s-a)(s-b)(s-c)}{F^3}}=\frac{3}{2R}\sqrt[3]{abc\cdot\frac{F^2}{sF^3}}=\frac{3}{2R}\sqrt[3]{\frac{abc}{sF}}=$$

$$=\frac{3}{2R}\cdot\sqrt[3]{\frac{4RF}{sF}}=\frac{3}{2R}\sqrt[3]{\frac{4R}{s}}\geq\frac{3}{2R}\sqrt[3]{4\cdot\frac{2}{3\sqrt{3}}\cdot\frac{s}{s}}=\frac{3}{2R}\cdot\frac{2}{\sqrt{3}}=\frac{\sqrt{3}}{R}$$

Equality holds for $a=b=c$