ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{\sin A}{r_a}+\frac{\sin B}{r_b}+\frac{\sin C}{r_c}\le\frac{\sqrt{3}}{2r}$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\frac{\sin A}{r_a}+\frac{\sin B}{r_b}+\frac{\sin C}{r_c}=\sum_{cyc}\frac{\sin A}{r_a}=\sum_{cyc}\frac{\frac{a}{2R}}{\frac{F}{s-a}}=$$

$$=\frac{1}{2RF}\sum_{cyc}a(s-a)=\frac{1}{2RF}\left(s\sum_{cyc}a-\sum_{cyc}a^2\right)=$$

$$=\frac{1}{2RF}(s\cdot 2s-2s^2+2r^2+8Rr)=\frac{2r(r+4R)}{2RF}=$$

$$=\frac{r(r+4R)}{Rrs}=\frac{r+4R}{Rs}\overset{EULER}{\le}\frac{\frac{R}{2}+4R}{Rs}=\frac{\frac{1}{2}+4}{s}\overset{MITRINOVIC}{\le}\frac{\frac{9}{2}}{3\sqrt{3}r}=\frac{3}{2\sqrt{3}r}=\frac{\sqrt{3}}{2r}$$

Equality holds for $a=b=c$.